Name _______ S O L U T I O N S _______
ECE Box # ____________

Problem Average Score Points

1 40.6 100

μ 40.6
σ 23.2
MED 31

ECE4902 B2015
Analog IC Design
Quiz 3
November 20, 2015

• This is a **closed book, closed notes test**! Use of calculators is OK, but **no pre-stored data or formulas allowed**!

• The last page is a notes page which may be detached for convenience and need not be turned in with the exam.

• Show **all** your work. Partial credit may be given. If you think you need something that you can't remember, write down what you need and what you'd do if you remembered it.

• **Look for the simple, straightforward way to solve the problem for the level of accuracy required.** Don't get entangled in unnecessary algebra.

• As in real life, some problems may give you more information than you need. Don't assume that all information must be used! It's your job to decide what's relevant to the solution.

• You have 40 minutes to complete this quiz. There is one problem on a total of 7 pages (including notes page).
1. It’s the night before the end of the term and you’ve got to get the ECE4902 lab reports handed in. But your lab partner had heard the professor’s “Confusing terminology = barrier to entry = high salary” line once too often and has dropped the course in frustration. Now all you have left is your sketchy lab notes for the common source amplifier with active load circuit shown below.

![Circuit Diagram]

You recall that you measured the step response of the circuit for two cases of the mirror resistor $R_B$. The table at the top of the opposite page shows what you have for each case:
- The value of $R_B$ and the voltage drop $V_B$ across $R_B$,
- The scope photo of the step response, showing the measured 10%-to-90% rise time
- The offset (MEAN) and peak-to-peak amplitude ($P_k-P_k$) of the waveform at the amplifier input, as well as the peak-to-peak amplitude ($P_k-P_k$) at the amplifier output.

For parts (a-c) you may ignore channel length modulation.

a) For each of cases I and II, determine the transfer function $v_{out}/v_{in}$ assuming the form of a first order lowpass. You may use $s$ or $f$ or $j\omega$, but be sure to express the transfer function in a form that explicitly shows the DC gain and the 3-dB frequency (bandwidth). Write your answers for cases I and II in the corresponding box on the opposite page. Accuracy $\pm 10\%$ for full credit. [30]

b) For each of case I and II, determine the small signal transconductance $g_{m1}$ of device M1. Write your answers for cases I and II in the corresponding box on the opposite page. Accuracy $\pm 10\%$ for full credit. [20]

c) Determine $\mu C_{ox}$ and $V_{th}$ for the NMOS and PMOS devices. Write your answers in the table on the opposite page. Accuracy for full credit: $\pm 10\%$ for $\mu C_{ox}$, $\pm 0.02V$ for $V_{th}$. [40]

d) You realize that these results can’t give you the individual channel length modulation parameters $\lambda_n$ and $\lambda_p$ for the NMOS and PMOS devices. But you can get the sum of the two. Determine the value of $\lambda_n + \lambda_p$ for the NMOS and PMOS devices. Write your answer in the space on the opposite page. Note that (since the $\lambda$ model for channel length modulation is the least accurate approximation to actual MOSFET behavior), you may get different values for $\lambda_n + \lambda_p$ depending on whether you use results from I or II. For the purposes of this problem, either one is OK. Accuracy for full credit: $\pm 20\%$. [10]
\( R_B \) = 35.8 \, \text{k}\Omega \\
\( V_B \) = 3.025 \, \text{V} \\
\( I_B = \frac{V_B}{R_B} \) = 84.5 \, \text{mA} \\
\( v_{out} = \frac{-36.8}{v_{in}} + j \left( \frac{f}{170 \, \text{kHz}} \right) \)

\( (b) \quad g_m = 334 \, \text{mA/V} \)

\[ v_{out} = \frac{-51.7}{v_{in}} + j \left( \frac{f}{5.67 \, \text{kHz}} \right) \]

\[ g_m = 157 \, \text{mA/V} \]

Answers for (c):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>N-channel</th>
<th>P-channel</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{TH} )</td>
<td>1.16 , \text{V}</td>
<td>-1.48 , \text{V}</td>
<td>\text{V}</td>
</tr>
<tr>
<td>( \mu \text{Cox} )</td>
<td>( 1.71 \times 10^{-5} , \frac{\text{A}}{\text{V}^2} )</td>
<td>( 7.66 \times 10^{-6} , \frac{\text{A}}{\text{V}^2} )</td>
<td>\text{A/V}^2</td>
</tr>
</tbody>
</table>

Answer for (d):

\[ \lambda_n + \lambda_p = 0.107 \, \text{or} \, 0.138 \]
a) DC GAIN FROM STEP AMPLITUDES (PK–PK)

\[
\frac{\Delta V_{out}}{\Delta V_{in}} = \frac{2.50V}{68mV} = 36.8 \quad \text{II}
\]

\[
\frac{2.48V}{48mV} = 51.7
\]

BANDWIDTH FROM \(BW \times t_r = 0.35\)

\[
\frac{0.35}{20.64\mu s} = 17.0\ \text{KHz} \quad \frac{0.35}{61.7\mu s} = 5.67\ \text{KHz}
\]

b) GAIN–BW PRODUCT \(f_T = \frac{g_m}{2\pi C_L}\)

\[
f_T = (36.8)(17.0\ \text{KHz})
\]

\[
= 626\ \text{KHz}
\]

\[
g_m = (626\ \text{KHz})2\pi (85\ \text{pF})
\]

\[
= 334\ \text{mA/V}
\]

\[
f_T = (51.7)(5.67\ \text{KHz})
\]

\[
= 293\ \text{KHz}
\]

\[
g_m = (293\ \text{KHz})2\pi (85\ \text{pF})
\]

\[
= 157\ \text{mA/V}
\]

NOTE: CAN CHECK THESE VS. RESULTS FROM (c)

AFTER (c) \(\approx 5\%\) DIFFERENCE, NOT BAD

\[
g_{m(I)} = \frac{2(84.5\ \text{mA})}{(1.69V - 1.16V)} = 319\ \text{mA/V}
\]

\[
g_{m(II)} = \frac{2(22\ \text{mA})}{(1.43V - 1.16V)} = 163\ \text{mA/V}
\]
c) WE HAVE 2 \( I_D-V_{GS} \) DATA POINTS WHICH WILL DETERMINE \( \mu_{COX} \) AND \( V_{TH} \)

\[ V_{GS}(DC) \] IS GIVEN BY MEAN OF INPUT CH2
IGNORING EFFECT OF \( \lambda V_{DS} \) WE ASSUME \( I_{D1}=I_{D3} \) (MIRROR)
SQUARE LAW FOR (I) AND (II)

\[(I) \quad 84.5\, \text{mA} = \frac{M_n\,C_{OX}}{2} \frac{350}{10} (1.69V - V_{TH(n)})^2 \]
\[(II) \quad 22\, \text{mA} = \frac{M_n\,C_{OX}}{2} \frac{350}{10} (1.43V - V_{TH(n)})^2 \]

DIVIDE EQUATIONS; TAKE SQUARE ROOT

\[ \frac{\sqrt{84.5\, \text{mA}}}{22\, \text{mA}} = \frac{1.69 V - V_{TH(n)}}{1.43 V - V_{TH(n)}} \Rightarrow 2.803V - 1.96 V_{TH(n)} = 1.69V - V_{TH(n)} \]

\[ \Rightarrow V_{TH(n)} = \frac{2.803V - 1.69V}{0.96} = 1.16\, V \]

SUB INTO EITHER (I) OR (II) TO FIND \( M_n\,C_{OX} \)

\[ 84.5\, \text{mA} = \frac{M_n\,C_{OX}}{2} \frac{350}{10} (1.69 - 1.16)^2 \Rightarrow M_n\,C_{OX} = 1.71 \times 10^{-5} \frac{A}{V^2} \]

\[ \text{PMOS} \]

SIMILARLY; \( V_{GS3} \) FROM KVL

\[(I) \quad 84.5\, \text{mA} = \frac{M_p\,C_{OX}}{2} \frac{900}{10} (-1.975V - V_{TH(p)})^2 \]
\[(II) \quad 22\, \text{mA} = \frac{M_p\,C_{OX}}{2} \frac{900}{10} (-1.732V - V_{TH(p)})^2 \]

\[ 1.96 = \frac{1.975 + V_{TH(p)}}{1.732 + V_{TH(p)}} \Rightarrow 3.395V + 1.96V_{TH(p)} = 1.975V + V_{TH(p)} \]

\[ V_{TH(p)} = \frac{1.975V - 3.395V}{0.96} = -1.48\, V \]

SUB FOR \( M_p\,C_{OX} \)

\[ 84.5\, \text{mA} = \frac{M_p\,C_{OX}}{2} \frac{900}{10} (-1.975 - (-1.48))^2 \Rightarrow M_p\,C_{OX} = 7.66 \times 10^{-6} \frac{A}{V^2} \]

\[ \times 4a \]
d) CAN GET FROM GAIN:

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = -g_m \frac{R_{\text{out}}}{(\lambda_n + \lambda_p)I_D}
\]

(\text{I})

\[
R_{\text{out}} = \frac{36.8}{334 \text{mA/V}} = 110.2 \text{k}\Omega
\]

\[
(\lambda_n + \lambda_p) = \frac{1}{(84.5 \text{mA})(110.2 \text{k}\Omega)} = 0.107
\]

(\text{II})

\[
R_{\text{out}} = \frac{51.7}{157 \text{mA/V}} = 329 \text{k}\Omega
\]

\[
(\lambda_n + \lambda_p) = \frac{1}{(22 \text{mA})(329 \text{k}\Omega)} = 0.138
\]

DIFFERENT SINCE CHARACTERISTICS DO NOT EXACTLY INTERSECT ON -V_{OS} AXIS

\[
\frac{-1}{0.107V^{'}} \quad \frac{-1}{0.138V^{'}}
\]
OR FOR (C) CAN USE \( \sqrt{I_D} \) VS \( V_{GS} \) METHOD

\[ \sqrt{I_{BSMA}} \rightarrow 9 \times 10^{-3} \]

\[ \sqrt{I_{ZMA}} \rightarrow 5 \times 10^{-3} \]

\( \sqrt{I_{KMA}} \rightarrow \)

NMOS

PMOS

\( V_{\text{TH(w)}} = 1.16 \text{V} \)

\(- V_{\text{TH(p)}} = 1.48 \text{V} \)

\[ \text{THEN SUB INTO SQUARE LAW FOR } \mu_{\text{Cox}} \]
\[ \sqrt{I_D} = \sqrt{\frac{\mu_{\text{Cox}}}{2} \frac{W}{L} (V_{GS} - V_{TH})} \]

**SLOPE**

**NMOS:** SLOPE IS \[ \frac{9 \times 10^{-3}}{0.52} = \sqrt{\frac{\mu_{n\text{Cox}}}{2}} \frac{350}{10} \]

\[ \mu_{n\text{Cox}} = 1.79 \times 10^{-5} \]

**PMOS:** \[ \frac{7.8 \times 10^{-3}}{0.42} = \sqrt{\frac{\mu_{p\text{Cox}}}{2}} \frac{900}{10} \]

\[ \mu_{p\text{Cox}} = 7.66 \times 10^{-6} \]
### MOSFET LARGE SIGNAL CHARACTERISTICS

#### P CHANNEL

<table>
<thead>
<tr>
<th>CIRCUIT SYMBOL</th>
<th>SATURATION REGION</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Circuit Diagram" /></td>
<td>$I_D = -\frac{\mu p C_{ox} W}{2L} (V_{GS} - V_{TH})^2 [1 + \lambda</td>
</tr>
</tbody>
</table>

#### N CHANNEL

<table>
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</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Circuit Diagram" /></td>
<td>$I_D = \frac{\mu n C_{ox} W}{2L} (V_{GS} - V_{TH})^2 [1 + \lambda V_{DS}]$</td>
</tr>
</tbody>
</table>

#### i_D-v_DS Characteristic (Saturation Region)

- **Saturation** ($|V_{DS}| > |V_{GS} - V_{TH}|$)
- **Triode** ($|V_{DS}| < |V_{GS} - V_{TH}|$)

#### i_D-v_GS Characteristic (Saturation Region)

- **Saturation** ($V_{GS} < V_{TH}$)
- **Cutoff** ($V_{GS} > V_{TH}$)

#### Triode Region

- **P Channel**
  
  $I_D = -\mu p C_{ox} \frac{W}{L} \left[ (V_{GS} - V_{TH})V_{DS} - \frac{V_{DS}^2}{2} \right]$ 

- **N Channel**
  
  $I_D = \mu n C_{ox} \frac{W}{L} \left[ (V_{GS} - V_{TH})V_{DS} - \frac{V_{DS}^2}{2} \right]$