Lecture 5 Review

• Current Source
• Active Load
• Modified Large / Small Signal Models
  – Channel Length Modulation

• Text sec 1.2 pp. 28-32; sec 3.2 pp. 128-129
Current source

- Ideal goal
- Small signal model:
  Open circuit
  "RD=∞"
Realizing current source: MOSFET

- Large signal nonideality: Compliance range
- “Looks like” current source only for $V_{DS} > V_{eff}$
MOSFET $I_D-V_{DS}$ characteristic for fixed $V_{GS}$

- Small-signal nonideality: slope in active region
Cause: Channel length modulation
Channel length modulation

$V_S = 0$

$V_{GS} = +3V$

$V_{DS} = +3V$

$\Delta L$

$\Delta V - V_{tn}$
Channel length modulation

$V_s = 0$

$V_{GS} = +3V$

$L_{eff}$

$V_{DS} = +4V$

$\Delta V - V_{tn}$

$\Delta L$

$V_{tn} = +1V$

$V_{GS} = +3V$

$V_{DS} = +4V$
Modify small-signal model: Finite $r_{ds}$ current source

- **Slope** = $\Delta I/\Delta V$

$$r_{ds} = \frac{\Delta V}{\Delta I}$$

$$SLOPE = \frac{\Delta I}{\Delta V} = \frac{1}{r_{ds}}$$

- **Ideal:**
  - zero slope
  - $r_{ds} = \infty$
Caution: $r_{DS(on)}$ vs. $r_{ds}$ confusion!

$r_{DS(on)}$
- Triode region
- Large signal
- True resistance (V-I through origin)

$r_{ds}$
- Active region
- Small signal
- Models nonideality of current source
Refined MOSFET Small Signal Model

- Add $r_{ds}$ in parallel with $g_m v_g$ current source at output
- SAME FOR N-ch, P-ch
- How to relate $r_{ds}$ to DC operating point?
- Example: $g_m = 2I_D/V_{eff}$
$I_D$- $V_{DS}$ Characteristic for Different $V_{GS}$

"Family" of curves
$I_D$ - $V_{DS}$ Characteristic for Different $V_{GS}$

Extrapolate backward: intersect at $V_{DS}$-axis
1/slope provides small signal resistance $r_{ds}$

- Intersect at $-1/\lambda$

Slope:

\[
\frac{1}{r_{ds}} = \frac{I_D}{1/\lambda} \quad r_{ds} = \frac{1}{\lambda I_D}
\]
MOSFET small signal model

\[ g_m = \frac{2I_D}{V_{eff}} \quad r_{ds} = \frac{1}{\Box I_D} \]
Increasing Gain

• Typical gain (resistive load)
  – Lab 4 example: $|a_v| \approx 2$
  – Class example: $|a_v| \approx 11.1$

• How to increase $a_v$?
Transconductance $g_m$

- **Definition**

\[ g_m = \frac{dI_D}{dV_{GS}} \]
Transconductance $g_m$

- **Definition**
  
  $$g_m = \frac{dI_D}{dV_{GS}}$$

- **$I_D$ from Square law:**
  
  $$\frac{dI_D}{dV_{GS}} = \frac{d}{dV_{GS}} nC_{ox} \frac{W_2}{L_2} \left(\frac{V_{GS}}{V_{eff}} - V_{tn}\right)^2$$

- **$g_m$ in terms of $W/L$, $V_{eff}$**
  
  $$g_m = nC_{ox} \frac{W_2}{L_2} V_{eff}$$
Summary of $g_m$ expressions

- All equivalent! choose whichever gives easier math
- Can’t memorize? rederive from definition of $g_m$

\[
g_m = \Box_n C_{ox} \frac{W_2}{L_2} V_{eff}
\]

\[
g_m = \frac{2I_D}{V_{eff}}
\]

\[
g_m = \sqrt{2\Box_n C_{ox} \frac{W}{L} I_D}
\]
Common source circuit (Lab 4)

Setting operating point:
Adjusted function generator offset for DC output at midpoint of signal swing
Common source circuit (Lab 4)

• DC operating point
• Chosen for “halfway” between rails
• $I_D=1.25\text{mA}$
• $V_{\text{eff}} \approx 2.0\text{V}$ (depends on parameters)

\[ V_{\text{OUT}} = V_{DD} - I_D R_D \]
\[ V_{\text{OUT}} = +2.5\text{V} \]
\[ I_D = \frac{V_{DD} - V_{\text{OUT}}}{R_D} \]
\[ = \frac{5\text{V} - 2.5\text{V}}{2\text{k}\Omega} = 1.25\text{mA} \]
Common source circuit (Lab 4)

- Small signal gain magnitude = 2.5
  
  \[ g_m = \frac{2I_D}{V_{eff}} = \frac{2(1.25\text{mA})}{2.0\text{V}} \]
  
  \[ g_m = 1.25\text{mA} / \text{V} \]
  
  \[ a_v = g_m R_D = (1.25\text{mA} / \text{V})(2 \text{ k}\Omega) \]
  
  \[ a_v = 2.5 \]

- Not impressive!
How to increase $a_v$?

Look at gain expression: $a_v = \left| g_m R_D \right|$
Increase $R_D$

- New $R_D = 10k\Omega$ (5X old value)
- Problem:
  - DC operating point
  - Violates condition for active region: triode!
- DC operating point stuck at negative rail

\[
V_{OUT} = V_{DD} \cdot I_D R_D
\]
\[
V_{OUT} = 5V \cdot \left(1.25mA\right) \left(10k\Omega\right) = 12.5V
\]
\[
V_{OUT} = \square7.5V?
\]
Look at problem symbolically

- Use $g_m = 2I_D/V_{eff}$

- $I_D R_D = $ DC drop on load

- Optimal bias at output: constrained to $V_{DD}/2$

- $I_D, R_D$ not involved!

- Value of approximate symbolic approach vs. “exact” numerical results from simulation
How to increase gain (resistive load)

• increase $V_{DD}$
  – usually fixed by application, process
• decrease $V_{eff}$
  – does increase $g_m$
  – but ...

$$|a_v| = \frac{V_{DD}}{V_{eff}}$$
Problems decreasing $V_{\text{eff}}$

- $V_{\text{eff}}$, W/L $g_m$ expression:
- 2X increase in $g_m$:
  - 4X increase in size
  - (can’t increase $I_D R_D$)
- Increased area:
  - cost penalty
- Increased capacitance
  - speed penalty
- $V_{\text{eff}} < \approx 200$ mV: subthreshold region
  - Not square law: $g_m$ expressions invalid

$$g_m = \sqrt{2 \Box_n C_{ox} \frac{W}{L} I_D}$$
Increase $a_v$: Different approach

• Give up on resistive load ...
• What is highest resistance?
Increasing $a_v$: Different approach

- What is highest resistance?
- Infinite: open circuit

Problem: no path for $I_D$

- Any circuit element that:
  – provides DC current, but
  – is open circuit in small signal model?
Current source!

- Open in small signal model
- Realizing current source: MOSFET
Lab Circuit: MOSFET with active load

- Small signal model for M1
- Thevenin equivalent “looking into” drain of M2 (see text sec. 3.1)
Small signal model

M1 common source

M2 Thevenin equivalent
Simplify small signal model

Combine $r_{ds1}$, $r_{ds2}$ in parallel
Small signal gain

\[
\frac{v_{out}}{v_{in}} = a_v = -g_{m1}(r_{ds1} || r_{ds2})
\]
Current source load: Large signal considerations

Output swing limits

- **Top:** M2 “crash” into triode
- **Bottom:** M1 crash into triode
Common Source with Active Load

- DC Sweep Schematic
Active Load Simulation Result (DC Sweep)

Top limit: $V_{DD} - V_{eff2}$

Bottom limit: $V_{SS} + V_{eff1}$
Determining DC Operating Point

Set small signal $v_{in} = 0$
Determining DC Operating Point

- **Active region:** 
  $V_{\text{eff}}$ determines $I_D$
- **Correct $V_{\text{IN}}$:** 
  M1, M2 “agree”
- **Example:** 
  $V_{\text{eff1}} = 1.0\, \text{V}$
  $I_D = 100\, \mu\text{A}$
If DC bias at input is “wrong”?

- Current source "disagreement"
- KCL crisis at output: 2µA, nowhere to go
- What happens?
If DC bias at input is “wrong”?

- Capacitance at output node $V_{out}$
- $2\mu A$ flows into cap, charges up
  - $V_{DS1}$ increases
  - $I_1$ increases
- $V_{DS2}$ decreases
  - $I_1$ decreases
- Changes in $V_{DS}$ cause changes in $I_D$ until “agreement” is reached: $I_{D1} = I_{D2}$
How much change in $V_{DS}$?

- Changes in $V_{DS}$ cause changes in $I_D$ until “agreement” is reached: $I_{D1} = I_{D2}$ BUT
- Active region: $I_D$ is a weak function of $V_{DS}$
- Large change in $V_{DS}$ for small change in $I_D$
- Output very sensitive to changes in $I_D$:
  - Small $\Delta V_{eff}$ at input $\Rightarrow$ Small $\Delta I_D$ $\Rightarrow$
  - Requires large $\Delta V_{DS}$ at output for $I_D$ agreement

**Good:** high voltage gain

**Bad:** tricky to get correct input bias point
Frequency Domain Considerations

• Ideal op-amp goals:
  – Infinite gain
  – Infinite bandwidth
• Active load helps gain
• What about bandwidth?
Frequency Domain Analysis

- Start simple: Assume single $C_L$ at output
- (Ignore MOS capacitances for now ...)
- Find transfer function $v_{out}/v_{in}$
- Combine $r_{ds1} || r_{ds2} = r_{out}$
Simpler small signal model

- Combine $r_{out} \ C_L$ into impedance $Z_L$
Simplified Small Signal Model

- Small signal gain: \( v_{out}/v_{in} = a_v = -g_m Z_L \)
- Frequency dependence of \( Z_L \) provides frequency dependence of transfer function
Closer Look at $Z_L$:

- Impedance is parallel combination of $r_{out}$, $1/sC_L$

$$Z_L = r_{out} \frac{1}{sC_L} = \frac{1}{\frac{1}{r_{out}} + sC_L}$$

$$Z_L = \frac{r_{out}}{1 + sr_{out}C_L}$$
Behavior of $Z_L$ over frequency:

- Let $s = j\omega$

  $$Z_L = \frac{r_{out}}{1 + j\omega r_{out}C_L}$$

- Low frequency limit: mostly $r_{out}$

  $$\omega \ll \frac{1}{r_{out}C_L} \quad Z_L = \frac{r_{out}}{1 + j\omega r_{out}C_L} \quad r_{out}$$

- High frequency limit: mostly $C_L$

  $$\omega \gg \frac{1}{r_{out}C_L} \quad Z_L = \frac{r_{out}}{1 + j\omega r_{out}C_L} \quad \frac{r_{out}}{j\omega r_{out}C_L} \quad \frac{1}{j\omega C_L}$$
Transfer function

• Substitute in $Z_L$

$$\frac{v_{out}}{v_{in}} = \square g_m Z_L = \frac{\square g_m r_{out}}{1 + \square j r_{out} C_L}$$

• Magnitude

$$\left| \frac{v_{out}}{v_{in}} \right| = \frac{g_m r_{out}}{\sqrt{1 + (\square r_{out} C_L)^2}}$$
Bode Plot of Transfer Function Magnitude

\[ \left| \frac{V_{out}}{V_{in}} \right| = \frac{g_m r_{out}}{\sqrt{1 + (\omega r_{out} C_L)^2}} \]

Bandwidth: \[ \Omega_{3dB} \]
3-dB Frequency / Bandwidth

- Frequency at which magnitude is 3 dB down (reduced by factor $1/\sqrt{2}$)

$$\max \left| \frac{v_{out}}{v_{in}} \right| = g_m r_{out} \quad AT \bigotimes = 0$$

THEN $AT \bigotimes_{3dB}, \left| \frac{v_{out}}{v_{in}} \right| = \frac{1}{\sqrt{2}} g_m r_{out}$

$$\frac{1}{\sqrt{2}} g_m r_{out} = \frac{g_m r_{out}}{\sqrt{1 + (\bigotimes_{3dB} r_{out} C_L)^2}} \bigotimes \bigotimes_{3dB} = \frac{1}{r_{out} C_L}$$
Revisit Bode Plot:

- Gain, Bandwidth inversely related!

\[ \omega_{3dB} = \frac{1}{r_{out} C_L} \]
Unity Gain Frequency $\square_T$ / Gain-Bandwidth Product

- $\square_T$: Frequency at which magnitude is 1
  Use approximation $\square_T \gg 1/r_{out}C_L$

$$1 = \frac{g_m r_{out}}{\sqrt{1 + (\square_T r_{out} C_L)^2}} \quad \frac{g_m r_{out}}{\sqrt{(\square_T r_{out} C_L)^2}} \quad \square_T = \frac{g_m}{C_L}$$

- Gain x Bandwidth Product

$$a_v = g_m r_{out} \quad \square_{3dB} = \frac{1}{r_{out} C_L} \quad \frac{g_m r_{out}}{GAIN} \cdot \frac{1}{r_{out} C_L} \quad = \frac{g_m}{C_L}$$

- Independent of $r_{out}$!
  - Poorly controlled $r_{out}$ OK
Summary: Active Load

• Active load DC considerations:
  – Output swing limited by triode “crash”
  – To voltage within $V_{\text{eff}}$ of rail

• Active load good news / bad news:
  – Good news: high gain
  – Bad news: very sensitive to input DC bias
Massage small signal gain result

- **Small signal gain**

$$|a_v| = g_m \left( \frac{1}{r_{ds1}} \parallel \frac{1}{r_{ds2}} \right)$$

$$r_{ds1} \parallel r_{ds2} = \frac{1}{\frac{1}{r_{ds1}} + \frac{1}{r_{ds2}}}$$

- **Look at parallel combination**

- **Substitute expression for** $r_{ds}$

$$r_{ds1} \parallel r_{ds2} = \frac{1}{\Box_1 I_D + \Box_2 I_D}$$

$$r_{ds1} \parallel r_{ds2} = \frac{1}{(\Box_1 + \Box_2) I_D}$$
Massage small signal gain result

• Small signal gain

\[ |a_v| = g_{m1} \left( r_{ds1} \parallel r_{ds2} \right) \]

• Substitute for \( g_m \), parallel \( r_{ds} \)

\[ |a_v| = \frac{2 I_D}{V_{eff1} \left( \square_1 + \square_2 \right) I_D} \frac{1}{g_{m1} r_{ds1} \parallel r_{ds2}} \]

\[ |a_v| = \frac{2}{\left( \square_1 + \square_2 \right) V_{eff1}} \]

Only \( \square, V_{eff} \) to work with
Improve Gain

• Reduce $V_{\text{eff}}$
  – Minimum $\approx$ 200 to 300mV (subthreshold)
  – May not want to go that low (W,L too big)

• Reduce $l_1$, $l_2$
  – How? Where does $l_1$ come from?
Square law model with channel length mod

\[
I_D = \frac{\mu_n C_{ox}}{2} \left( \frac{W}{L} (V_{GS} - V_{tn}) \right)^2 \left[ 1 + \lambda \left( V_{DS} - V_{eff} \right) \right]
\]

\[I_D \text{-sat term (at pinchoff) + "extra"}\]
Square law model with channel length modulation

\[ I_D = \frac{\Box n C_{ox}}{2} \frac{W}{L} (V_{GS} \Box V_{tn})^2 \left[ 1 + \Box (V_{DS} \Box V_{eff}) \right] \]

- Fractional extra part is \( \Box (V_{DS} - V_{eff}) \)
- Meaning of \( \Box \):
  Fractional change in current \( I_D \) per volt change in \( V_{DS} \)
What causes change? Where does \( L \) come from?

- Change in effective channel length \( L \)
- One way to reduce \( L \): longer \( L \)
- Change \( \Delta L \) represents smaller fraction
After some semiconductor physics ...

- Definition of \( \square \)
  \[
  \square = \frac{I_D/I_D}{V_{DS}}
  \]

- Fractional change
  \[
  \frac{I_D}{I_D} = \frac{L}{L}
  \]
  \[
  \square = \frac{1}{L} \frac{L}{V_{DS}}
  \]

- Semiconductor physics ...
  (see J&M p. 26)

- \( K_S \) Silicon dielectric constant 11.8
- \( N_{SUB} \) Substrate doping units /cm\(^3\)
  Sanity check: 1E+14 to 1E+17
- \( \Delta V_{DS} \) from active-triode edge to “large” \( V_{DS} \)
- Caution: consistent length units on \( L, N_{SUB}, \square_0 \)
Substrate doping NSUB parameter

- Needed for SPICE

Extraction procedure:

1) Calculate slope from $I_D$-$V_{DS}$ plot
2) $r_{ds} = 1/\text{slope} \ (\text{small signal model})$
3) Calculate $[]$
4) Calculate NSUB
Example

$V_{DS}$-$I_D$ data from Lab 5 for P-channel MOSFET:

SLOPE IN ACTIVE REGION = $1/r_{ds}$

$r_{ds} = \frac{1.93V}{48 \mu A} = 40.2 \text{ kQ}$
1) Calculate slope from $I_D-V_{DS}$ plot
2) $r_{ds} = 1/$slope (small signal model)
3) Calculate \( \omega \)

\[
\omega = \frac{1}{I_D \cdot r_{ds}} = \frac{1}{(482 \ \mu A)(40.2 \ k\Omega)} = \frac{1}{19.4 \ V} = 0.052 \ V^{-1}
\]
4) calculate NSUB

\[ \Theta = \frac{1}{L \sqrt{\frac{2K_S \rho_0}{qN_{SUB}}} \frac{1}{2\sqrt{V_{DS} \cdot V_{eff}}}} \]

\[
0.052V^{\frac{1}{1}} = \frac{1}{(1E \cdot 5m)} \sqrt{\frac{2(11.8)(8.85E \cdot 12F/m)}{(1.6E \cdot 19coul)N_{SUB}}} \frac{1}{2\sqrt{4.48V \cdot 0.84V}} 
\]

\[ N_{SUB} = 3.32E + 22 \text{ m}^3 = 3.32E + 16 \text{ cm}^3 \]

- For CD4007, \( L = 10\mu m = 1.0E-5m \)
- \( V_{DS}, V_{eff} \) for largest \( V_{DS} \) data point
Simulation exercise

- Add NSUB to N-channel, P-channel models
- DC sweep for CS Amplifier with Active Load
Common Source with Active Load (DC)

- Sweep input over full range 0 to +5V
DC Sweep Around Operating Point

DC Response

\( e = \frac{\text{Vout}}{V} \)

\( dc \, (V) \):

- A
- B

Voltage Range:
- 0.0 to 5.0

Voltage Axis:
- 0.0 to 5.0

DC Voltage Range:
- 2.0 to 3.0

Graph shows the relationship between the output voltage (Vout) and the dc voltage (V) around a specific operating point.