### N-Channel MOSFET Operating Regions

<table>
<thead>
<tr>
<th>Gate</th>
<th>Drain</th>
<th>Region</th>
<th>First Order Behavior</th>
<th>Not Exactly:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{GS} &gt; V_{TH}$</td>
<td>$V_{DS} &gt; V_{GS} - V_{TH}$</td>
<td>Saturation (Active)</td>
<td>Drain &quot;Looks Like&quot; Current Source; $I_D$ Depends Only on $V_{GS} - V_{TH}$ $I_D = \frac{\mu_n C_{ox} W}{2L} (V_{GS} - V_{TH})^2$</td>
<td>Channel Length Modulation $I_D$ Depends Somewhat on $V_{DS}$ $I_D = \frac{\mu_n C_{ox} W}{2L} (V_{GS} - V_{TH})^2 [1 + \lambda V_{DS}]$</td>
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<td>$V_{GS} &lt; V_{TH}$</td>
<td>$V_{DS} &lt; V_{GS} - V_{TH}$</td>
<td>Triode</td>
<td>D-S Channel &quot;Looks&quot; Resistive $I_D = V_{DS} / R_{on}$ $R_{on}$ Depends on $V_{GS} - V_{TH}$</td>
<td>Nonlinear as $V_{DS}$ Increases $I_D = \frac{\mu_n C_{ox} W}{L} \left[ (V_{GS} - V_{TH}) V_{DS} - \frac{V_{DS}^2}{2} \right]$</td>
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<tr>
<td>$V_{GS} &lt; V_{TH}$</td>
<td>$</td>
<td>V_{DS}</td>
<td>&lt; V_{th}$</td>
<td>Cutoff</td>
</tr>
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**Diagram:**
- **Id** vs. $V_{GS}$ and $V_{DS}$
- **Active** vs. **Triode** vs. **Cutoff**
- **Increasing** $V_{GS}$
- **Transconductance** $\frac{dI_D}{dV_{GS}}$ vs. $V_{GS}$
- **Breakdown** if $|V_{DS}|$ Exceeds $V_{th}$
## P-Channel MOSFET Operating Regions

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<th>REGION</th>
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<td>$V_{DS} &lt; V_{GS} - V_{TH}$</td>
<td>SATURATION (ACTIVE)</td>
<td>DRAIN &quot;LOOKS LIKE&quot; CURRENT SOURCE; $I_D$ DEPENDS ONLY ON $V_{GS} - V_{TH}$</td>
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<td>$V_{GS} &lt; V_{TH}$</td>
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<td>TRIODE</td>
<td>$I_D = \frac{\mu_p C_{ox} W}{2 L} (V_{GS} - V_{TH})^2$</td>
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</tr>
<tr>
<td></td>
<td>$</td>
<td>V_{DS}</td>
<td>&lt; V_{bkd}$</td>
<td>CUTOFF</td>
</tr>
</tbody>
</table>

**Diagrams:**
- ** Define $+I_D$**
- **Negative MORE SATURATION (ACTIVE)**
- **Negative TRANSISTOR**
- **Negative TRANSCONDUCTANCE**
- **Negative SLOPE**
- **Negative VTH**
- **Negative ID**
- **Negative VDS**
- **Negative VGS**
3) [MOSFET operating region practice]

For each MOSFET below, determine the MOSFET operating region (active, triode, cutoff). Assume NMOS $V_{TH} = +1V$ and PMOS $V_{TH} = -1V$.

a)

$V_G = +3V$
$V_GS = +3V$
$V_D = +5V$
$V_DS = +5V$

$V_{GS} > V_{TH} ?$
$+3V > +1V$ (CUT OFF)

$V_{DS} < V_{GS} - V_{TH} ?$
$+5V < 3V - 1V$
$2V$

$V_{DS} > V_{GS} - V_{TH}$

(b)
4) [More MOSFET operating region practice]

For each MOSFET below, determine

a) the MOSFET operating region (active, triode, cutoff), and
b) the drain voltage $V_D$ and the DC drain current $I_D$.

Assume NMOS $V_{TH} = +1\text{V}$ and PMOS $V_{TH} = -1\text{V}$.

For both NMOS and PMOS, assume $\mu C_{ox} \frac{W}{L} = 2.0E-4 \frac{A}{V^2}$

\[
\text{a)} \quad V_{GS} \rightarrow I_D \rightarrow V_D = V_{DD} - I_D R_D
\]

\[
R_D = \frac{V_{DD}}{I_D} = \frac{100k\Omega}{100mA} = 1k\Omega
\]

\[
I_D = \frac{V_{GS} - V_{TH}}{R_D} = \frac{V_{GS} - 1V}{1k\Omega} = \frac{3V - 1V}{1k\Omega} = 2A
\]

\[
V_D = V_{DD} - I_D R_D = 5V - 2A \times 1k\Omega = 3V
\]

\[
I_D = \frac{V_{GS} - V_{TH}}{R_D} = \frac{1A}{1k\Omega} = 1A
\]

\[
V_D = V_{DD} - I_D R_D = 5V - 1A \times 1k\Omega = 4V
\]

\[
\text{b)} \quad V_{GS} \rightarrow I_D \rightarrow V_D = V_{DD} - I_D R_D
\]

\[
R_D = \frac{V_{DD}}{I_D} = \frac{100k\Omega}{100mA} = 1k\Omega
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\]

\[
\text{c)} \quad V_{GS} \rightarrow I_D \rightarrow V_D = V_{DD} - I_D R_D
\]

\[
R_D = \frac{V_{DD}}{I_D} = \frac{100k\Omega}{100mA} = 1k\Omega
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I_D = \frac{V_{GS} - V_{TH}}{R_D} = \frac{V_{GS} - 1V}{1k\Omega} = \frac{3V - 1V}{1k\Omega} = 2A
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\]
Sec. 3.2 Common-Source Stage

1. $V_{IN} = 0$ INCREASING
   $V_{IN} < V_{TH}$: CUTOFF: $I_D = 0$
   $V_{OUT} = V_{DD}$

2. $V_{GS}$ A LITTLE $> V_{TH}$ $V_{GS} - V_{TH}$ SMALL
   $V_{DS} \approx V_{DD}$ BIG $\leftarrow$ ACTIVE
   $V_{OUT} = V_{DD} - \frac{M_{COX} W}{2} \left( V_{IN} - V_{TH} \right)^2 R_D$
   $I_D$ FROM $S$ & LAW

3. $V_{DS} \leq (V_{GS} - V_{TH})$ "TRIODE CRASH"

AMPLIFIER!? WANT: $V_{OUT}$

$V_{OUT} = A V_{IN}$

HOW WELL DOES CS DO AMPLIFIER??

1. CUTOFF
2. ACTIVE
3. TRIODE

LINEAR APPROXIMATION "small signal"

CALL THIS "ZERO" "OPERATING POINT"

BIGGER OUTPUT CHANGE
$\Delta V_{OUT}$ SLOPE: "small signal gain"

Small input change
$V_{IN}$

$\Delta V_{IN}$
INCREMENTAL (SMALL SIGNAL) ANALYSIS

What it is
A method of analysis that allows us to get approximate analytic expressions (equations) for nonlinear circuits which can't be solved easily.
Concept: "Take derivative first"

Why you do it
Linear signal analysis is such a powerful tool, we're going to use it to analyze systems that aren't linear. Anything (even a nonlinear circuit element) looks linear if you look at small enough changes from an operating point.

PROCEDURE:

1. First, find the DC (large signal) operating point for each element in the nonlinear circuit.

   Possible methods:
   - Solve nonlinear equations (e.g. quadratic for active region MOSFET square-law model)
   - Iteratively solve nonlinear equations (e.g. SPICE)
   - Approximate analysis (e.g. for BJT, assume $V_{BE} = 0.7V$ in active region)
   - Graphical technique

Small signal solution will "ride on" bias levels provided by large signal operating point solution

2. Redraw the circuit: replace each circuit element with its small-signal model
   - Linear elements (e.g. pure R, L, C) stay the same
   - Constant V/I Sources: Gone! ("take derivative first"):  
     - DC voltage sources: replace with short circuit
     - DC current sources: replace with open circuit
   - Nonlinear element: Replace with small-signal model
     - For each type of device, small-signal model is obtained by taking derivative of appropriate terminal characteristic to find linear approximation for behavior around operating point.
     - Usually just do this once for each type of device; small signal model parameters are a function of large signal operating point (e.g. small signal MOSFET model derived once; then for each application of model use operating point information to calculate small signal parameters).
3. Solve the small-signal circuit model using all the linear analysis tools you know and love:

**Good Old Ohm's Law**
All V-I characteristics are linear in small-signal model

**Can use Thevenin's theorem to simplify large circuits**
Attack them one block at a time
Helps to understand functions of each block; how well actual circuit is performing function

**Can use superposition to calculate response to different inputs**
Attack output behavior one signal at a time
Helps to understand response (output) as caused by each input

**Can use transfer functions to express frequency-dependent behavior**

**Can always use KVL, KCL, nodal analysis**
(apply to any circuit, linear or nonlinear)

4. Total behavior is sum of DC (large signal) operating point + small signal component "riding on" DC bias from large signal operating point solution

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**Cautions**

**Limitations**
If actual signal isn't "small", then "solution" won't be valid!
How small is "small"? Depends on accuracy required. Need to look at derivation of model for individual devices used.

**Common errors**
Don't get large signal, small signal quantities confused! For example:
There should be no DC sources (e.g. supply rails) in a small signal model.
SMALL SIGNAL ANALYSIS SUMMARY

CIRCUIT

\[ V_{DD} = +5V \]
\[ R_D = 10k\Omega \]
\[ V_{IN} = +1.06V \]

MOSFET:
\[ \mu C_{ox} = 7.42E-5 \, \text{A/V}^2 \]
\[ V_t = 0.61V \]

LARGE SIGNAL MODEL

\[ V_{DD} = +5V \]
\[ V_{IN} = +1.06V \]
\[ V_{OSS} = 8.0 \, \text{V} \]
\[ V_{GS} = \text{SQUARE LAW} \]
\[ I_D = I_{Vgs} \]

SMALL SIGNAL MODEL

\[ V_{OSS} = 2.4V \]
\[ V_{OUT} = i_d R_D \]

DC VOLTAGES
\[ \text{SIGNAL GROUND} \]

SIGNAL GROUND

\[ i_d = g_m V_{gs} \]

TRANCONDUCTANCE \( g_m \) FROM DC BIAS
\[ g_m = 7.42E-5 \, \frac{80}{2.4} \, (1.06 - 0.61) \]
\[ g_m = 1.11 \, \text{mA/V} \]

\[ V_{OUT} = -i_d R_D = -\frac{g_m V_{gs}}{V_{IN}} R_D = \frac{(-g_m R_D) V_{IN}}{V_{IN}} \]
\[ V_{OUT} = -11.1 \]

SMALL SIGNAL GAIN
\[ \frac{V_{OUT}}{V_{IN}} = -g_m R_D \]

\[ V_{OUT} = +2.49 \, \text{V} \]
\[ 2.49V > 0.45V \]

OVERDRIVE

\[ V_{DS} > V_{eff} \, \text{ACTIVE} \]
"BUILD UP" OUTPUT FROM DC, SMALL SIGNAL "PIECES"

AT INPUT:

\[
\frac{V_{IN\text{ TOTAL}}}{V_{IN}} = +1.06\,V + (0.1\,V) \sin \omega t
\]

\[
\frac{V_{OUT\text{ TOTAL}}}{V_{OUT}} = +2.49\,V + (-1.11\,V) \sin \omega t
\]

OUTPUT \downarrow \quad \downarrow \text{CHANGE} \quad \downarrow \text{PROPORTIONAL GAIN}

\[
V_{OUT} = \frac{(gmR_o)V_{IN}}{-11.1}
\]

FROM SMALL-SIGNAL ANALYSIS

IN GENERAL

\[
\frac{V_{OUT\text{ TOTAL}}}{V_{OUT}} = \frac{V_{OUT\text{ DC OPERATING POINT}}}{V_{OUT\text{ SMALL SMALL SIGNAL GAIN}}} + A_v\frac{V_{IN\text{ SMALL SIGNAL INPUT}}}{V_{IN}}
\]

SKETCH \( V_{OUT} \)

\[+5\,V \quad +3.60\,V \quad +2.49\,V \quad +1.38\,V \quad 0\,V \]

\[t \]

CHECK MAX, MIN PEAKS:

MEETS ACTIVE REGION CONDITIONS FOR ALL \( V_{OUT} \)

WHAT IF \( V_{IN} = (1.0\,V) \sin \omega t \)? SMALL SIGNAL MODEL PREDICTS

\[
V_{OUT} = 2.49 + (-11.1) \sin \omega t
\]

CUTOFF \( \leftrightarrow \) MAX PEAK = 13.6 \, V ?

CHECK PEAK VALUES

TRIODE \( \leftrightarrow \) MIN PEAK = -8.6 \, V ?
\[ g_m = \frac{dI_D}{dV_{GS}} \]

\[ I_D = \frac{\mu C_0 W}{2} \left( V_{GS} - V_{TH} \right) \]

\[ \frac{dI_D}{dV_{GS}} = \mu C_0 \frac{W}{L} \left( V_{GS} - V_{TH} \right) = g_m \]