Labs 8 & 9 Review

• Operational Amplifier
  – Stability
  – Compensation
  – Miller Effect
  – Phase Margin
  – Unity Gain Frequency
  – Slew Rate Limiting

• Reading: Razavi ch. 9, 10
  – Lab 8, 9 op-amp is Fig. 10.34 in sec. 10.5.1
  – (see also Johns & Martin sec 5.2 pp. 232-242)
Two-stage op-amp

$V_{DD} = +5V$

All P: $\frac{900}{10}$

All N: $\frac{350}{10}$

$V_{in}$

$V_{out}$

$50\mu A$

$V_{GS}$

$V_{SS} = -5V$
Analysis Strategy

- Recognize sub-blocks
- Represent as cascade of simple stages
Total op-amp model

Input differential pair      Common source stage
## DC operating point

<table>
<thead>
<tr>
<th></th>
<th>$I_D [\mu A]$</th>
<th>$V_{GS-V_{TH}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>25</td>
<td>0.235</td>
</tr>
<tr>
<td>M2</td>
<td>25</td>
<td>0.235</td>
</tr>
<tr>
<td>M3</td>
<td>25</td>
<td>0.247</td>
</tr>
<tr>
<td>M4</td>
<td>25</td>
<td>0.247</td>
</tr>
<tr>
<td>M5</td>
<td>50</td>
<td>0.350</td>
</tr>
<tr>
<td>M6</td>
<td>50</td>
<td>0.332</td>
</tr>
<tr>
<td>M7</td>
<td>50</td>
<td>0.332</td>
</tr>
<tr>
<td>M8</td>
<td>50</td>
<td>0.332</td>
</tr>
</tbody>
</table>
## Small signal parameters

<table>
<thead>
<tr>
<th></th>
<th>$I_D[\mu A]$</th>
<th>$V_{GS-V_{TH}}$</th>
<th>$g_m[\mu A/V]$</th>
<th>$r_O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>25</td>
<td>0.235</td>
<td>208</td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>25</td>
<td>0.235</td>
<td></td>
<td>800kΩ</td>
</tr>
<tr>
<td>M3</td>
<td>25</td>
<td>0.247</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M4</td>
<td>25</td>
<td>0.247</td>
<td></td>
<td>1.43MΩ</td>
</tr>
<tr>
<td>M5</td>
<td>50</td>
<td>0.350</td>
<td>285</td>
<td>715kΩ</td>
</tr>
<tr>
<td>M6</td>
<td>50</td>
<td>0.332</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M7</td>
<td>50</td>
<td>0.332</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M8</td>
<td>50</td>
<td>0.332</td>
<td></td>
<td>400kΩ</td>
</tr>
</tbody>
</table>

Note: $\lambda_n = 0.050 \text{ V}^{-1}$; $\lambda_p = 0.028 \text{ V}^{-1}$
Total op-amp model: Low frequency gain

Input differential pair

\[ a_{v1} = g_{m1} \left( r_{o2} || r_{o4} \right) \]
\[ a_{v1} = (208 \mu A/V)(800k\Omega || 1.43M\Omega) \]
\[ a_{v1} = 106 \]

Common source stage

\[ a_{v2} = g_{m2} \left( r_{o5} || r_{o8} \right) \]
\[ a_{v2} = (285 \mu A/V)(400k\Omega || 715k\Omega) \]
\[ a_{v2} = 73 \]
Total op-amp model with capacitances

Gate of M5  
Load: scope probe $\approx 10\text{pF}$

$$C_g = (900 \, \mu\text{m})(10\, \mu\text{m}) \left(4.17E - 4 \frac{F}{m^2}\right)$$

$$C_g = 3.74 \, \text{pF}$$
Total op-amp model with capacitances

First stage pole

\[ f_{p1} = \frac{1}{2\pi (r_{O2}||r_{O4})C_{g5}} \]

\[ f_{p1} = \frac{1}{2\pi (800k\Omega||1.43M\Omega)(3.74\,pF)} \]

\[ f_{p1} = 82kHz \]

Second stage pole

\[ f_{p1} = \frac{1}{2\pi (r_{O5}||r_{O8})C_L} \]

\[ f_{p1} = \frac{1}{2\pi (400k\Omega||715k\Omega)(10\,pF)} \]

\[ f_{p1} = 61kHz \]
Open loop transfer function

• Product of individual stage transfer functions

\[
A(j\omega) = \frac{g_{m1}(r_{o2}\|r_{o4}) g_{m5}(r_{o5}\|r_{o8})}{\left[1 + j\omega(r_{o2}\|r_{o4})C_{g5}\right]\left[1 + j\omega(r_{o5}\|r_{o8})C_L\right]}
\]

• Numerically (using \( \omega = 2\pi f \))

\[
A(j\omega) = \frac{7738}{\left[1 + j\left(\frac{f}{82kHz}\right)\right]\left[1 + j\left(\frac{f}{61kHz}\right)\right]}
\]

• Check Bode plot simulation; predicts:
  – DC gain = 20log(7738) = +78dB
  – Unity gain frequency \( \sim 6.2 \text{ MHz} \)
Two-stage op-amp: Simulation Schematic
DC Operating Point Simulation

DC Response

OP POINT 2.885 mV

Systematic Offset!

A: (2.11783m -3.81929) delta: (1.39772m 7.02858)
B: (3.51555m 3.20929) slope: 5.02881K
Bode plot

- Magnitude, phase on log scales
- Pole: Root of denominator polynomial

SLOPE:
-20dB/dec
Open loop Bode plot

- Product of terms: Sum on log-log plot

-20 dB/dec

-40 dB/dec

-90°

-180°
Open Loop Bode Plot Simulation

Note: AC source at input also needs DC component to account for systematic offset!
Check Open Loop Bode Plot Simulation

DC gain \( \sim +78 \text{dB} \)

Unity gain \( \sim 16 \text{MHz} \)
Stability example: Closed loop follower

- **Negative feedback:**
  Output connected to inverting input
- **Gain should be ~ 1**

\[
\begin{align*}
    v_{out} &= A(v_{in} - v_{out}) \\
    v_{out}(A + 1) &= Av_{in} \\
    v_{out} &= \left( \frac{A}{A + 1} \right) v_{in} \\
    &\approx 1 \text{ as } A \gg 1
\end{align*}
\]
Unity gain: Why bother?

- No buffer: Voltage divider
- Signal reduced due to voltage drop across $R_S$

$$v_{out} = \left( \frac{R_L}{R_L + R_S} \right) v_{in}$$

- With buffer: No current required from source

$$v_{out} = v_{in}$$
Lab 9 Problem: Instability

- Oscillation superimposed on desired output!??
Lab 9 Problem: Instability

- Ground $v_{in}$: Output for zero input?!?
- Why? Need...

\[ v_{IN} \]

\[ v_{OUT} \]
Controls: ECE3012 in 20 minutes

• General framework
  A: Forward Gain
  $\beta$: Feedback Factor
  fraction of output fed back to input
Example: Op-amp, Noninverting Gain

**A: Forward Gain**

Op-amp open loop gain

\[ V_{out} = A(V_+ - V_-) \]

Transfer function \( A(j\omega) \)

**\( \beta \): Feedback Factor**

\[ \beta = \frac{R_1}{R_1 + R_2} \]
Closed Loop Gain

- **Output**
  \[ v_{out} = A \left( v_{in} - \beta v_{out} \right) \]
  \[ v_{out} = A v_{in} - A \beta v_{out} \]

- **Solve for** \( v_{out}/v_{in} \)
  \[ (1 + A\beta) v_{out} = A v_{in} \]
  \[ \frac{v_{out}}{v_{in}} = \frac{A}{1 + A\beta} \]
Op-amp with negative feedback

- If $A\beta >> 1$

$$\frac{v_{out}}{v_{in}} = \frac{A}{1 + A\beta} \approx \frac{A}{A\beta} \Rightarrow \frac{v_{out}}{v_{in}} \approx \frac{1}{\beta}$$

- Closed loop gain determined only by $\beta$

- Advantage of negative feedback:
  Open loop gain $A$ can be ugly (nonlinear, poorly controlled) as long as it's large!
Example: Op-amp, Noninverting Gain

\( \beta: \text{Feedback Factor} \)

\[
\beta = \frac{R_1}{R_1 + R_2}
\]

Closed loop gain

\[
\frac{v_{out}}{v_{in}} = \frac{R_1 + R_2}{R_1} = \frac{1}{\beta}
\]
Reexamine closed loop transfer function

- Output with no input: infinite gain
  \[ \frac{v_{out}}{v_{in}} = \frac{A}{1 + A\beta} \]

- Infinite when \( 1 + A\beta = 0 \)

- Condition for oscillation:
  \[ 1 + A\beta = 0 \]

- In general \( A, \beta \) functions of \( \omega \)

- If there's a frequency \( \omega \) at which \( 1 + A\beta = 0 \):
  Oscillation at that frequency!
Example: follower

\[ \beta = 1 \quad \rightarrow \quad \frac{v_{out}}{v_{in}} = \frac{A}{1 + A} \]

- Use \( A(j\omega) \), solve for \( 1 + A = 0 \)
- No thanks!

\[
A(j\omega) = \frac{g_{m1}(r_{O2}||r_{O4})g_{m5}(r_{O5}||r_{O8})}{\left[1 + j\omega(r_{O2}||r_{O4})C_{g5}\right]\left[1 + j\omega(r_{O5}||r_{O8})C_L\right]} 
\]
Reexamine condition for oscillation

\[ 1 + A\beta = 0 \rightarrow A\beta = -1 \]

Magnitude and phase condition:

\[ |A\beta| = 1 \ \text{AND} \ \angle A\beta = -180^\circ \]

- Easier to get from Bode plot
Look at original $A\beta$ for 2 stage op-amp

- Find $\omega$ at which $|A\beta| = 1$; Check $\angle A\beta \ -180^\circ$?

Trouble!
Simulation $A\beta$ for 2 stage op-amp

AC Response

Unity loop gain at $\sim 16$MHz

$> 180^\circ$ phase lag at unity loop gain!

- Causes closed-loop instability
**Compensation: “Dominant Pole”**

- **Move one pole to lower frequency**
- **How?**

**Move unity loop gain frequency** $f_T$ **to lower value**

So accumulated phase lag at $f_T$ hasn’t reached -180°
Compensation: “Dominant Pole”

- Need to increase capacitance by $\approx 1000X$:

BAD! Die area cost
Miller Effect

- Impedance across inverting gain stage $G$
- Reduced by factor equal to $(1+G)$
Math for Miller effect

\[ i_x = \frac{v_x - (-Gv_x)}{Z} \]

\[ i_x = \frac{v_x(1 + G)}{Z} \]

\[ \frac{v_x}{i_x} = Z_{in} = \frac{Z}{(1 + G)} \]

- Impedance across inverting gain stage G
- Reduced by factor equal to \((1+G)\)
Example: Impedance is capacitive

- Capacitance multiplied by (1+G)

\[
Z_{in} = \frac{Z}{1 + G}
\]

\[
Z = \frac{1}{sC} \quad \Rightarrow \quad Z_{in} = \frac{1}{s(1 + G)C}
\]

- Equivalent capacitance higher by factor 1+G
- Problem for high bandwidth amplifiers
- Opportunity for compensation ...
Miller Compensation

- Need effect of large capacitance
- Use Miller effect to multiply small on-chip capacitance to higher effective value
- Effect of large capacitance without die area cost of large capacitance
New schematic

• Add $C_C$ across 2nd stage
New loop gain transfer function

Unity loop gain at ~65kHz

125° phase lag at unity loop gain
New step response

- No oscillation!

\[ v_{IN} \]

\[ v_{OUT} \]
New step response with $C_C$

- Zoom in on small-signal step response:
  Some overshoot and ringing
Reason: RHP zero in complete transfer function

Complete transfer function looks like:

\[ A(j\omega) = \frac{A_0 \left[1 - j(\omega/\omega_z)\right]}{\left[1 + j(\omega/\omega_{p1})\right]\left[1 + j(\omega/\omega_{p2})\right]} \]

Open loop gain \( A \) with only 2 poles

Effect of RHP zero: additional phase lag

See Razavi 10.5, Johns & Martin 5.2
"Phase margin"

- How stable is new transfer function?
- Phase margin = Phase lag at $|A\beta| = 1$ minus (-180°)
- Usually want at least 60° for stable step response
Phase margin of op-amp with $C_C$

AC Response

Unity loop gain at ~65kHz

125° phase lag at unity loop gain

Phase margin = 55°
Solution to RHP zero problem

- Add $R_Z$ in series with $C_C$
  Moves RHP zero to much higher frequency
New step response with $R_Z, C_C$

- Zoom in on small-signal step response:
  No overshoot, ringing: phase margin improved
Large signal step response

• Slew Rate Limiting!?!?

See Solomon op-amp paper for model; rising/falling asymmetry
Dominant pole op-amp model

- Simpler model with dominant pole from $C_C$
Approximate dominant pole transfer function

\[ |A(j\omega)| \approx \frac{g_{m1}(r_{o2}\parallel r_{o4})A_2}{1 + j\omega(r_{o2}\parallel r_{o4})A_2C_C} \]

\[ A_2 = g_{m5}(r_{o5}\parallel r_{o8}) \]

Miller multiplied

2\textsuperscript{nd} stage gain

\[ C_C \]
Unity gain frequency

- Depends only on
  - Input stage transconductance $g_{m1}$
  - Compensation capacitor $C_C$

\[
|A(j\omega)| \approx \frac{g_{m1}(r_{o2}\|r_{o4})A_2}{\omega(r_{o2}\|r_{o4})A_2C_C}
\]

\[
|A(j\omega)| = 1 \quad \text{at} \quad \omega_T
\]

\[
\omega_T \approx \frac{g_{m1}}{C_C}
\]
Slew rate

- $I = C \frac{dV}{dt}$
- Only limited current $I_{BIAS}$ available to charge, discharge $C_C$
Slew rate

- $I = C \frac{dV}{dt} \Rightarrow \frac{dV}{dt} = \frac{I_{BIAS}}{C_C}$
Summary Op-amp:

- Stability
- Compensation
- Miller effect
- Phase Margin
- Unity gain frequency
- Slew Rate Limiting