Studio 4 Review

- Common Source Amplifier, Resistive Load
- Small Signal Modeling
## Review operating regions: N-channel MOSFET

<table>
<thead>
<tr>
<th>GATE</th>
<th>DRAIN</th>
<th>REGION</th>
<th>FIRST ORDER BEHAVIOR</th>
<th>NOT EXACTLY:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{GS} &gt; V_{TH} )</td>
<td>( V_{DS} &gt; V_{GS} - V_{TH} )</td>
<td>SATURATION</td>
<td>DRAIN &quot;LOOKS LIKE&quot; CURRENT SOURCE; ( I_D ) DEPENDS ONLY ON ( V_{GS} - V_{TH} )</td>
<td>CHANNEL LENGTH MODULATION</td>
</tr>
<tr>
<td>( V_{GS} &gt; V_{TH} )</td>
<td>( V_{DS} &lt; V_{GS} - V_{TH} )</td>
<td>TRIODE</td>
<td>D-S CHANNEL &quot;LOOKS&quot; RESISTIVE ( I_D = V_{DS} / R_{on} ) ( R_{on} ) DEPENDS ON ( V_{GS} - V_{TH} )</td>
<td>NONLINEAR AS ( V_{DS} ) INCREASES</td>
</tr>
<tr>
<td>( V_{GS} &lt; V_{TH} )</td>
<td>(</td>
<td>V_{DS}</td>
<td>&lt; V_{bkd} )</td>
<td>CUT OFF</td>
</tr>
</tbody>
</table>
Review MOSFET: Vary $V_{DS}$ ($V_{GS}$ fixed)

- Need $V_{GS} > V_{TH}$ (avoid cutoff region)
Review MOSFET: Vary $V_{GS}$ ($V_{DS}$ fixed)

- Need $V_{DS} > V_{GS} - V_{TH}$ (avoid triode region)
Common Source Amplifier

- $V_{IN}$ goes up $\Rightarrow$
  - Increase $V_{GS}$ $\Rightarrow$
  - Increase $I_D$
- KVL path for $V_{out}$
  - $V_{out} = V_{DD} - I_D R_D$
- MOSFET in saturation:
- $I_D$ increases $\Rightarrow$
  - $I_D R_D$ drop increases $\Rightarrow$
  - $V_{OUT}$ goes down

\[
V_{IN} \text{ goes up } \Rightarrow \quad V_{OUT} \text{ goes down}
\]
Common Source Amplifier

• INVERTING GAIN

GATE ⇒ DRAIN

$V_{IN}$ goes up ⇒ $V_{OUT}$ goes down
Build up input-output characteristic

- $V_{IN} < V_{TH} \Rightarrow V_{GS} < V_{TH} \Rightarrow$
  - Cutoff: $I_D = 0$ $V_{OUT} = V_{DD}$
Build up input-output characteristic

- \( V_{IN} > V_{TH} \) \( V_{GS} > V_{TH} \) \( V_{OUT} = V_{DD} - \frac{\mu_n C_{ox} W}{L} (V_{in} - V_{TH})^2 R_D \)

- Saturation: \( I_D \) from square law
Condition for saturation-triode boundary

- $V_{DS} V_{GS} - V_{TH} \Rightarrow$
- On boundary, both triode, saturation equations apply
Condition for saturation-triode boundary

- Expression for $V_{\text{out}}$ on boundary (use square law)

\[
V_{DD} - \frac{\mu_n C_{ox} W}{2} \frac{L}{V_{\text{out}}} (V_{in} - V_{TH})^2 R_D = V_{in} - V_{TH}
\]

- Ugly quadratic: solve numerically or let SPICE do it
Triode

• Ugly math: let SPICE do the work
• Minimum output \( \approx V_{GS} - V_{TH} \) (gate overdrive!)
• Maximize output swing \( \Leftrightarrow \) minimize overdrive
Input-output characteristic

• Compare to desired amplifier characteristic …
Desired linear amplifier characteristic

\[ v_{out} = a_v \cdot v_{in} \]  (negative slope OK - but should be linear!)
Linear (sort of) part of input-output characteristic

- Key: Small signal $\Rightarrow$ Slope = Derivative
Slope: derivative

• Find slope (small signal gain)

\[ \frac{dV_{OUT}}{dV_{in}} = \frac{d}{dV_{in}} \left[ V_{DD} - \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{in} - V_{TH})^2 R_D \right] \]

\[ \frac{dV_{OUT}}{dV_{in}} = -\mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH}) R_D \]

• Constant terms (such as supply $V_{DD}$) drop out

• Why carry constant terms through equations if they're just going to drop out when we take derivative? ...
Small Signal Analysis

• What it is
  – A method of analysis that allows us to get approximate analytic expressions (equations) for nonlinear circuits which can't be solved easily.
  – "Take derivative first"
Small Signal Analysis: Why you do it

• Linear signal analysis: powerful tool
• Use it to analyze systems that aren't linear.
• Anything (even a nonlinear circuit element) looks linear if you look at small enough changes from an operating point.
Small Signal Analysis: Procedure:

1. First, find the DC (large signal) operating point for each element in the nonlinear circuit.

2. Redraw the circuit: replace each circuit element with its small-signal model

3. Solve the small-signal circuit model using all the linear analysis tools you know and love

4. Total behavior is sum of DC (large signal) operating point + small signal component "riding on" DC bias from large signal operating point solution
Small Signal Analysis: Procedure:

1. First, find the DC (large signal) operating point for each element in the nonlinear circuit.

   – Possible methods:
     - Solve nonlinear equations (e.g. quadratic for saturation region MOSFET square-law model)
     - Iteratively solve nonlinear equations (e.g. SPICE)
     - Approximate analysis (e.g. for diode, assume $V_D = 0.7V$ when forward biased)
     - Graphical technique

   – Small signal solution will "ride on" bias levels provided by large signal operating point solution
Small Signal Analysis: Procedure:

2. Redraw the circuit: replace each circuit element with its small-signal model
   
   – Linear elements (e.g. pure R, L, C) stay the same

   – Constant V/I Sources: Gone!
     ("take derivative first"): 
     • DC voltage sources: replace with short circuit
     • DC current sources: replace with open circuit
Small Signal Analysis: Procedure:

- Nonlinear element: Replace with small-signal model
  - For each type of device, small-signal model is obtained from derivative of appropriate terminal characteristic to find linear approximation for behavior around operating point.
  - Usually just do this once for each type of device; small signal model parameters are a function of large signal operating point (e.g. small signal MOSFET model derived once; then for each application of model use operating point information to calculate small signal parameters).
Small Signal Analysis: Procedure:

3. Solve the small-signal circuit model using all the linear analysis tools you know and love:

   – Good Old Ohm's Law
   – Can use Thevenin's theorem to simplify large circuits
   – Can use superposition to calculate response to different inputs
   – Can use transfer functions to express frequency-dependent behavior
   – Can always use KVL, KCL, nodal analysis (these don't require linearity)
Small Signal Analysis: Procedure:

4. Total behavior is sum of DC (large signal) operating point + small signal component "riding on" DC bias from large signal operating point solution
Small Signal Analysis: Limitations

- If actual signal isn't "small", then "solution" won't be valid!
- How small is "small"? Depends on accuracy required. Need to look at derivation of model for individual devices used.
- ALWAYS check output waveform max/min to be sure assumptions for linear model are valid (for example, output not clipped at supply rails)
Simplest MOSFET Small Signal Model

- Saturation Region: Transconductance $g_m$

$$I_D = \frac{\mu_n C_{ox} W}{2} \frac{W}{L} (V_{GS} - V_{TH})^2$$

$$\frac{dI_D}{dV_{GS}} = g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

![MOSFET Diagram]

![Transconductance Graph]
Summary

- MOSFET operating regions
- Common source amplifier
- Large signal input-output characteristic
- Small-signal concept
Lab exercise

• Plot Large Signal input-output characteristic
• Measure small-signal gain