A	70 - 100
B	50 - 69
C	30 - 49

μ 40.6

σ 23.2

MED 31

Name SOLUTIONS

ECE Box # ____________

Problem	Average Score	Points
1	40.6	100

ECE4902 B2015

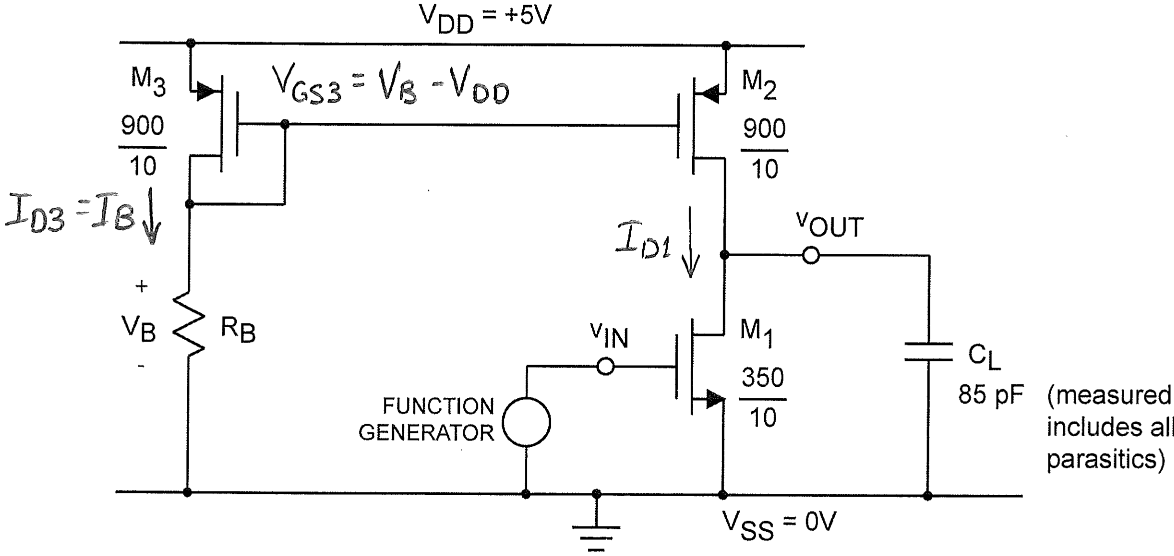
Analog IC Design

Quiz 3

November 20, 2015

- This is a **closed book, closed notes test**! Use of calculators is OK, but **no pre-stored data or formulas allowed**!
- The last page is a notes page which may be detached for convenience and need not be turned in with the exam
- Show **all** your work. Partial credit may be given. If you think you need something that you can't remember, write down what you need and what you'd do if you remembered it.
- **Look for the simple, straightforward way to solve the problem for the level of accuracy required. Don't get entangled in unnecessary algebra.**
- As in real life, some problems may give you more information than you need. Don't assume that all information must be used! It's your job to decide what's relevant to the solution.
- You have 40 minutes to complete this quiz. There is one problem on a total of 7 pages (including notes page).

1. It's the night before the end of the term and you've got to get the ECE4902 lab reports handed in. But your lab partner had heard the professor's "Confusing terminology = barrier to entry = high salary" line once too often and has dropped the course in frustration. Now all you have left is your sketchy lab notes for the common source amplifier with active load circuit shown below.



You recall that you measured the step response of the circuit for two cases of the mirror resistor R_B. The table at the top of the opposite page shows what you have for each case:

- The value of R_B and the voltage drop V_B across R_B,
- The scope photo of the step response, showing the measured 10%-to-90% rise time
- The offset (MEAN) and peak-to-peak amplitude (Pk-Pk) of the waveform at the amplifier input, as well as the peak-to-peak amplitude (Pk-Pk) at the amplifier output.

For parts (a-c) you may ignore channel length modulation.

a) For each of cases I and II, determine the transfer function v_{out}/v_{in} assuming the form of a first order lowpass. You may use *s* or *f* or *j*ω, but be sure to express the transfer function in a form that explicitly shows the DC gain and the 3-dB frequency (bandwidth). Write your answers for cases I and II in the corresponding box on the opposite page. Accuracy ±10% for full credit. **[30]**

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b) For each of case I and II, determine the small signal transconductance g_{m1} of device M1. Write your answers for cases I and II in the corresponding box on the opposite page. Accuracy ±10% for full credit. **[20]**

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c) Determine μC_{OX} and V_{TH} for the NMOS and PMOS devices. Write your answers in the table on the opposite page. Accuracy for full credit: ±10% for μC_{OX}, ± 0.02V for V_{TH}. **[40]**

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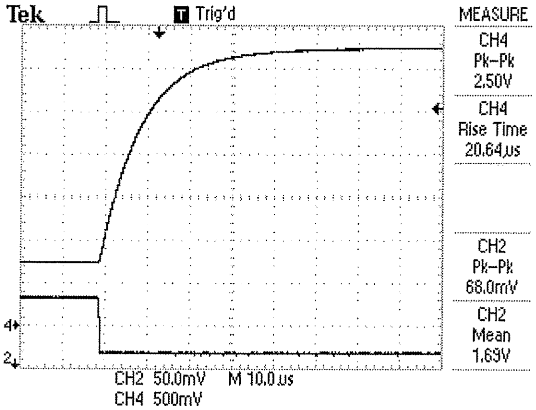
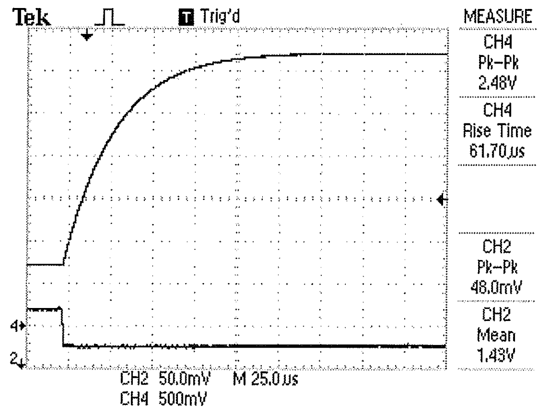
d) You realize that these results can't give you the individual channel length modulation parameters λ_n and λ_p for the NMOS and PMOS devices. But you can get the sum of the two. Determine the value of $\lambda_n + \lambda_p$ for the NMOS and PMOS devices. Write your answer in the space on the opposite page. Note that (since the λ model for channel length modulation is the least accurate approximation to actual MOSFET behavior), you may get different values for $\lambda_n + \lambda_p$ depending on whether you use results from I or II. For the purposes of this problem, either one is OK. Accuracy for full credit: ±20%. **[10]**

I	II

CH4
VOUT
500mV/div

CH2
VIN
50mV/div





	I	II
R_B	35.8 kΩ	148.7 kΩ
V_B	3.025 V	3.268 V
$I_B = \frac{V_B}{R_B}$	84.5 µA	22.0 µA

(a)

I: $$\frac{v_{out}}{v_{in}} = \frac{-36.8}{1 + j\left(\frac{f}{170\,kHz}\right)}$$

II: $$\frac{v_{out}}{v_{in}} = \frac{-51.7}{1 + j\left(\frac{f}{5.67\,kHz}\right)}$$

(b)

I: $g_m =$ 334 µA/V

II: $g_m =$ 157 µA/V

Answers for (c):

Parameter	N-channel	P-channel	Units
V_{TH}	1.16 V	−1.48 V	V
μC_{ox}	$1.71\,E\text{-}5\ \frac{A}{V^2}$	$7.66\,E\text{-}6\ \frac{A}{V^2}$	A/V^2

Answer for (d):

$\lambda_n + \lambda_p =$ 0.107 or 0.138

a) DC GAIN FROM STEP AMPLITUDES (PK-PK)

I

$\frac{\Delta V_{OUT}}{\Delta V_{IN}} = \frac{2.50V}{68\,mV} = 36.8$

II

$\frac{2.48V}{48\,mV} = 51.7$

BANDWIDTH FROM $BW \times t_R = 0.35$

$\frac{0.35}{20.64\,\mu S} = 17.0\,KHz$

$\frac{0.35}{61.7\,\mu S} = 5.67\,KHz$

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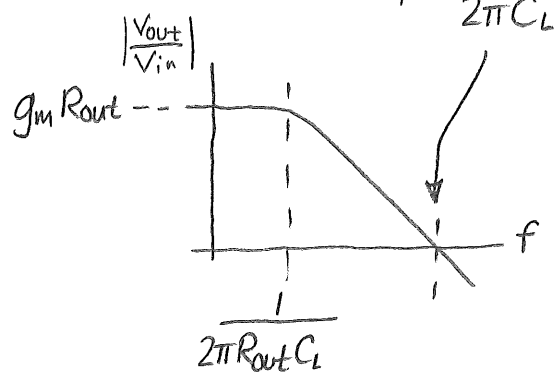
b) GAIN-BW PRODUCT $f_T = \frac{g_m}{2\pi C_L}$



I

$f_T = (36.8)(17.0\,kHz)$

$= 626\,kHz$

$g_m = (626\,kHz)\,2\pi\,(85\,pF)$

$= 334\,\mu A/V$

II

$f_T = (51.7)(5.67\,kHz)$

$= 293\,kHz$

$g_m = (293\,kHz)\,2\pi\,(85\,pF)$

$= 157\,\mu A/V$

NOTE: CAN CHECK THESE VS. RESULTS FROM (c) AFTER (c)

$\simeq 5\%$ DIFFERENCE. NOT BAD

$g_{m(I)} \overset{?}{=} \frac{2(84.5\,\mu A)}{(1.69V - 1.16V)} = 319\,\frac{\mu A}{V}$

$g_{m(II)} \overset{?}{=} \frac{2(22\,\mu A)}{(1.43V - 1.16V)} = 163\,\frac{\mu A}{V}$

c) WE HAVE 2 I_D-V_{GS} DATA POINTS WHICH WILL DETERMINE μC_{ox} AND V_{TH}

NMOS

$V_{GS(DC)}$ IS GIVEN BY MEAN OF INPUT CH2

IGNORING EFFECT OF λV_{DS} WE ASSUME $I_{D1} = I_{D3}$ (MIRROR)

SQUARE LAW FOR (I) AND (II)

$$\text{(I)} \quad 84.5\mu A = \frac{\mu_n C_{ox}}{2} \frac{350}{10} \left(1.69V - V_{TH(N)}\right)^2$$

$$\text{(II)} \quad 22\,\mu A = \frac{\mu_n C_{ox}}{2} \frac{350}{10} \left(1.43V - V_{TH(N)}\right)^2$$

DIVIDE EQUATIONS; TAKE SQUARE ROOT

$$\underbrace{\sqrt{\frac{84.5\,\mu A}{22\,\mu A}}}_{1.96} = \frac{1.69\,V - V_{TH(N)}}{1.43\,V - V_{TH(N)}} \Rightarrow 2.803V - 1.96\,V_{TH(N)} = 1.69V - V_{TH(N)}$$

$$\Rightarrow V_{TH(N)} = \frac{2.803V - 1.69V}{0.96} = \underline{1.16\,V}$$

SUB INTO EITHER (I) OR (II) TO FIND $\mu_n C_{ox}$

$$84.5\mu A = \frac{\mu_n C_{ox}}{2} \frac{350}{10} \left(1.69 - 1.16\right)^2 \Rightarrow \mu_n C_{ox} = 1.71E\text{-}5 \ \frac{A}{V^2}$$

PMOS SIMILARLY; V_{GS3} FROM KVL

$$\text{(I)} \quad 84.5\mu A = \frac{\mu_p C_{ox}}{2} \frac{900}{10} \left(-1.975\,V - V_{TH(P)}\right)^2$$

$$22\,\mu A = \frac{\mu_p C_{ox}}{2} \frac{900}{10} \left(-1.732V - V_{TH(P)}\right)^2$$

$$1.96 = \frac{1.975 + V_{TH(P)}}{1.732 + V_{TH(P)}} \Rightarrow 3.395V + 1.96\,V_{TH(P)} = 1.975V + V_{TH(P)}$$

$$V_{TH(P)} = \frac{1.975V - 3.395V}{0.96} = -1.48\,V$$

SUB FOR $\mu_p C_{ox}$

$$84.5\mu A = \frac{\mu_p C_{ox}}{2} \frac{900}{10} \left(-1.975 - (-1.48)\right)^2 \Rightarrow \mu_p C_{ox} = 7.66\,E\text{-}6 \ \frac{A}{V^2}$$

d) CAN GET FROM GAIN:

$$\frac{V_{out}}{V_{in}} = -g_m \underbrace{R_{out}}_{\frac{1}{(\lambda_n+\lambda_p) I_D}}$$

(I)

$$R_{out} = \frac{36.8}{334\,\mu A/V} = 110.2\,k\Omega$$

$$(\lambda_n+\lambda_p) = \frac{1}{(84.5\,\mu A)(110.2\,k\Omega)}$$

$$= 0.107$$

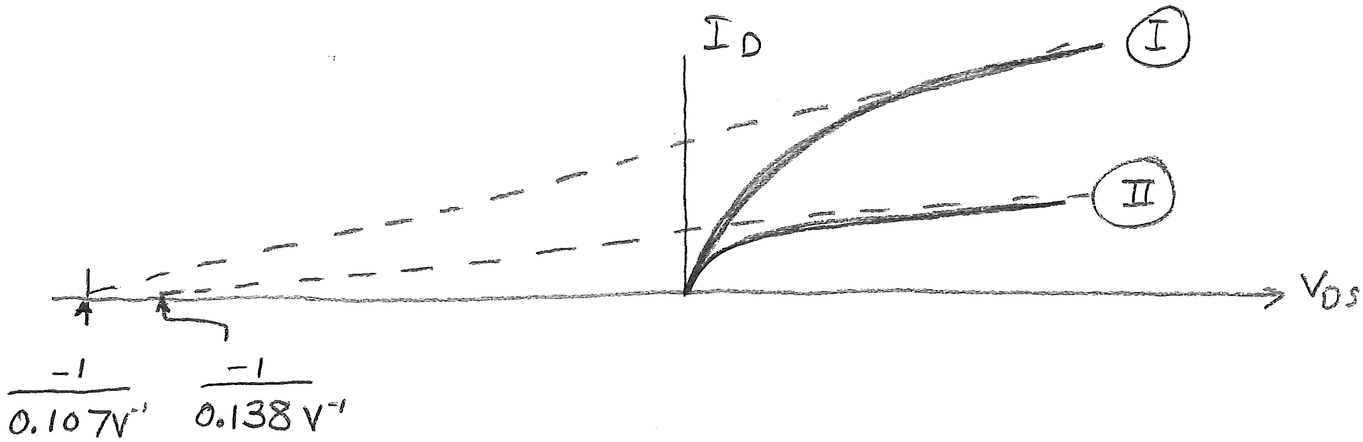
(II)

$$R_{out} = \frac{51.7}{157\,\mu A/V} = 329\,k\Omega$$

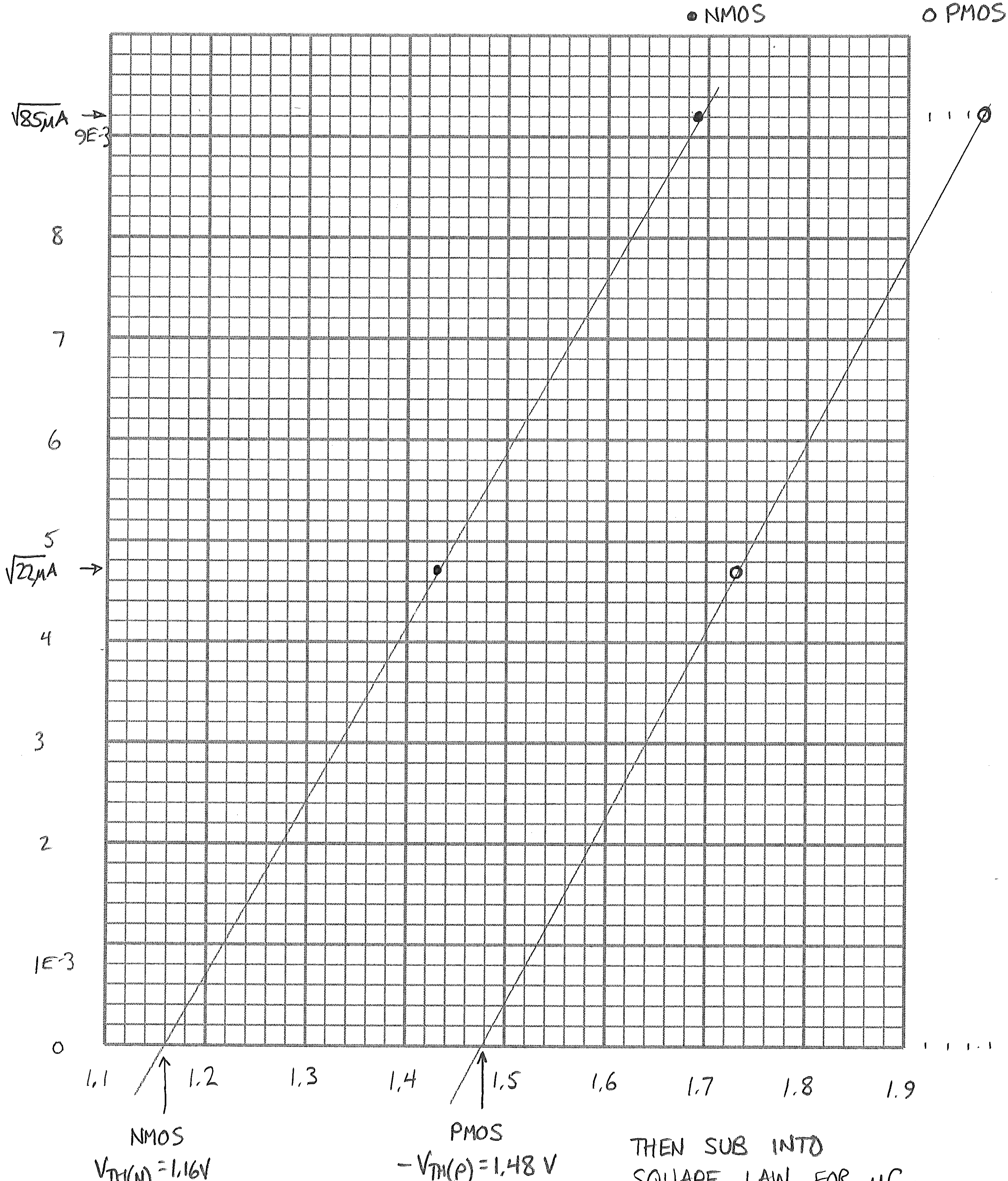
$$(\lambda_n+\lambda_p) = \frac{1}{(22\,\mu A)(329\,k\Omega)}$$

$$0.138$$

DIFFERENT SINCE CHARACTERISTICS DO NOT EXACTLY INTERSECT ON $-V_{DS}$ AXIS



OR FOR (C) CAN USE $\sqrt{I_D}$ VS V_{GS} METHOD



THEN SUB INTO
SQUARE LAW FOR μC_{ox}

$\sqrt{I_D}$ METHOD:

$$\sqrt{I_D} = \underbrace{\sqrt{\frac{\mu C_{ox}}{2}\frac{W}{L}}}_{\text{SLOPE}}\left(V_{GS} - V_{TH}\right)$$

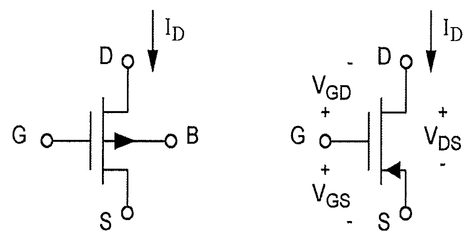
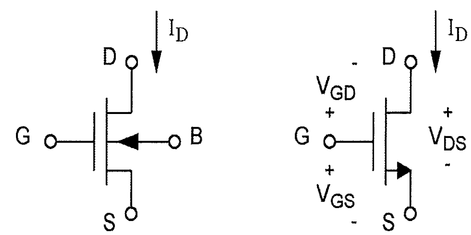
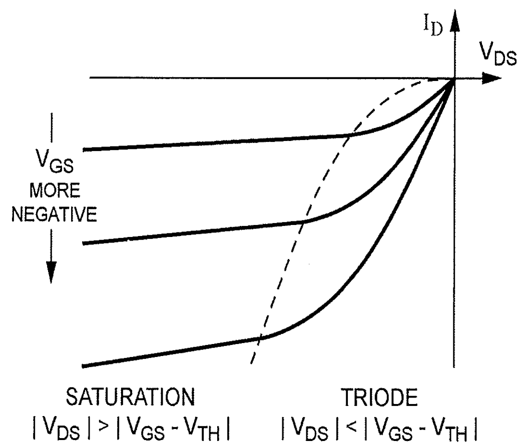
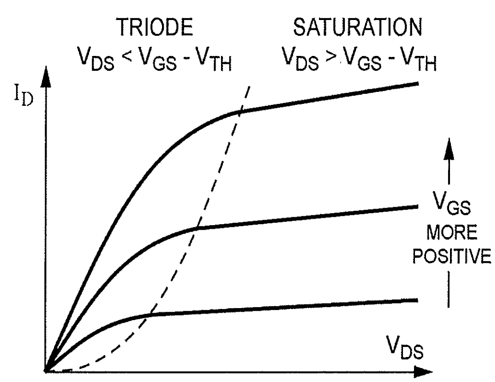
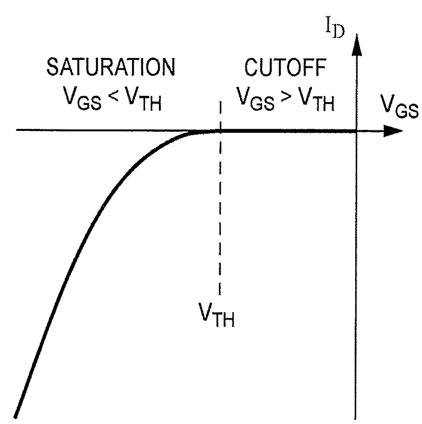
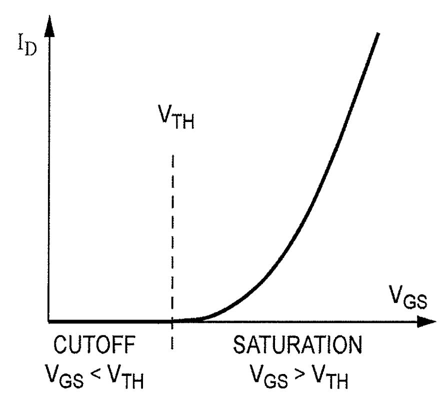
NMOS: SLOPE IS $\overbrace{\frac{9E\text{-}3}{0.52}}^{\frac{RISE}{RUN}} = \sqrt{\frac{\mu_n C_{ox}}{2}\frac{350}{10}}$

$$\mu_n C_{ox} = 1.79E\text{-}5$$

PMOS: $\frac{7.8E\text{-}3}{0.42} = \sqrt{\frac{\mu_p C_{ox}}{2}\frac{900}{10}}$

$$\mu_p C_{ox} = 7.66E\text{-}6$$

MOSFET LARGE SIGNAL CHARACTERISTICS

	P CHANNEL	N CHANNEL
CIRCUIT SYMBOL		
SATURATION REGION	$I_D = -\frac{\mu_p C_{ox}}{2}\frac{W}{L}(V_{GS}-V_{TH})^2\left[1+\lambda\lvert V_{DS}\rvert\right]$	$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_{TH})^2\left[1+\lambda V_{DS}\right]$
I_D-V_{DS} CHARACTERISTIC		
I_D-V_{GS} CHARACTERISTIC (SATURATION REGION)		
TRIODE REGION	$I_D = -\mu_p C_{ox}\frac{W}{L}\left[(V_{GS}-V_{TH})V_{DS} - \frac{V_{DS}^2}{2}\right]$	$I_D = \mu_n C_{ox}\frac{W}{L}\left[(V_{GS}-V_{TH})V_{DS} - \frac{V_{DS}^2}{2}\right]$