ECE4902 Lecture 9

2-Stage Op-Amp
- 1st-2nd Stage: NMOS or PMOS?!?
- DC Analysis
- Input Common-Mode Range
- Feedback
- Stability

Handouts:

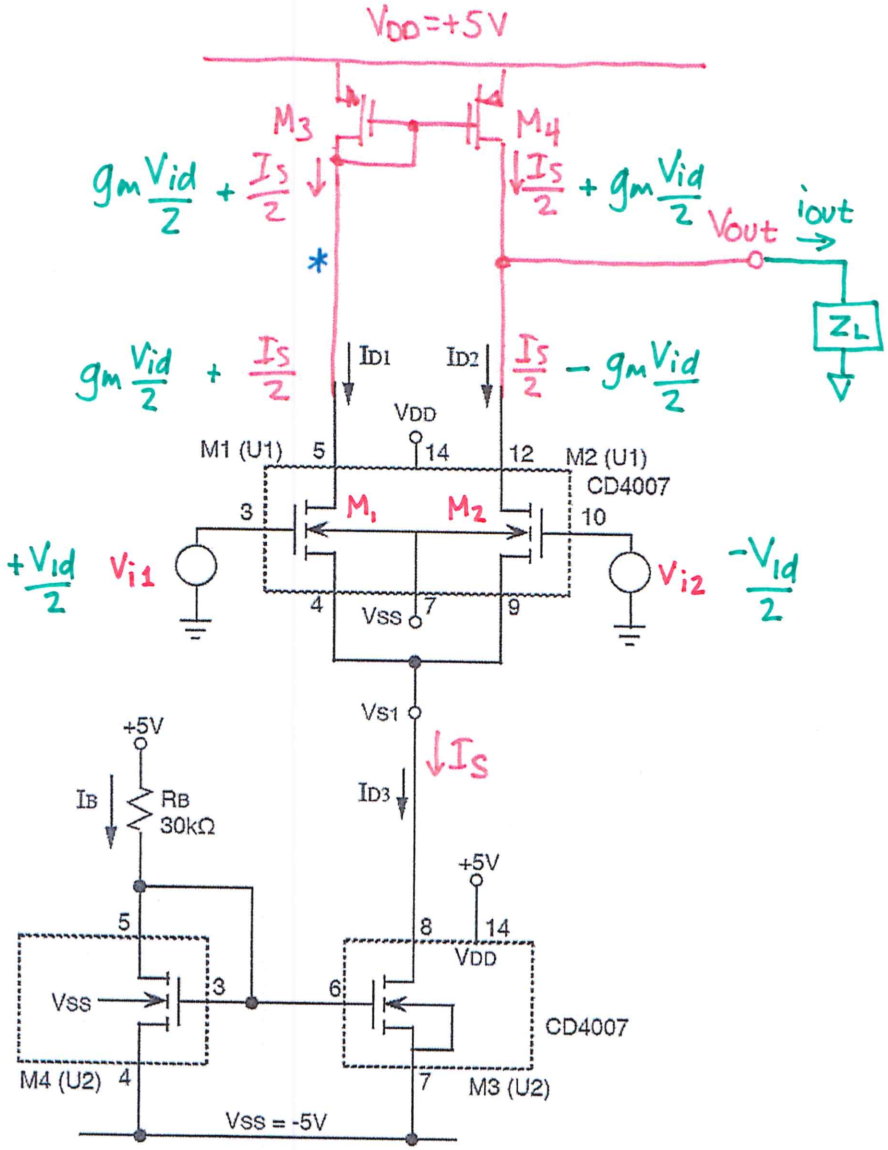
Lab 8/9 Review Slides

Differential Pair
2nd Stage Common Source
Lab 8 Op-Amp

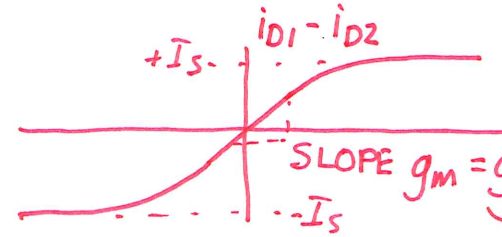
THE MOST IMPORTANT LECTURE OF ~~ECE4902~~ !

YOUR LIFE

Differential Pair with Mirror Load: Single Ended Output; Recovering "Other" Half of Differential Signal



DIFF PAIR TRANSCONDUCTANCE (½ CIRCUIT)



$V_{i1} - V_{i2} = V_{id}$

SLOPE $\underbrace{g_m = g_{m1} = g_{m2}}_{\text{DC BIAS CONDITION}}$

$I_{D1} = I_{D2} = \frac{I_S}{2}$

SIGNAL: $V_{id} = V_{i1} - V_{i2}$

SYMMETRY: HALF AT EACH INPUT

M1: $\frac{+V_{id}}{2} \Rightarrow I_{D1} \uparrow$ HOW MUCH? $\frac{\Delta I_D}{\Delta V_{GS}} = g_m$

M2: $\frac{-V_{id}}{2} \Rightarrow I_{D2} \downarrow$

WHAT ABOUT M3, M4 MIRROR?

M3: SAME AS M1 I_{D1}

M4: MIRROR I_{D3} TO I_{D4}

KCL "DISAGREEMENT" AT Vout NODE!

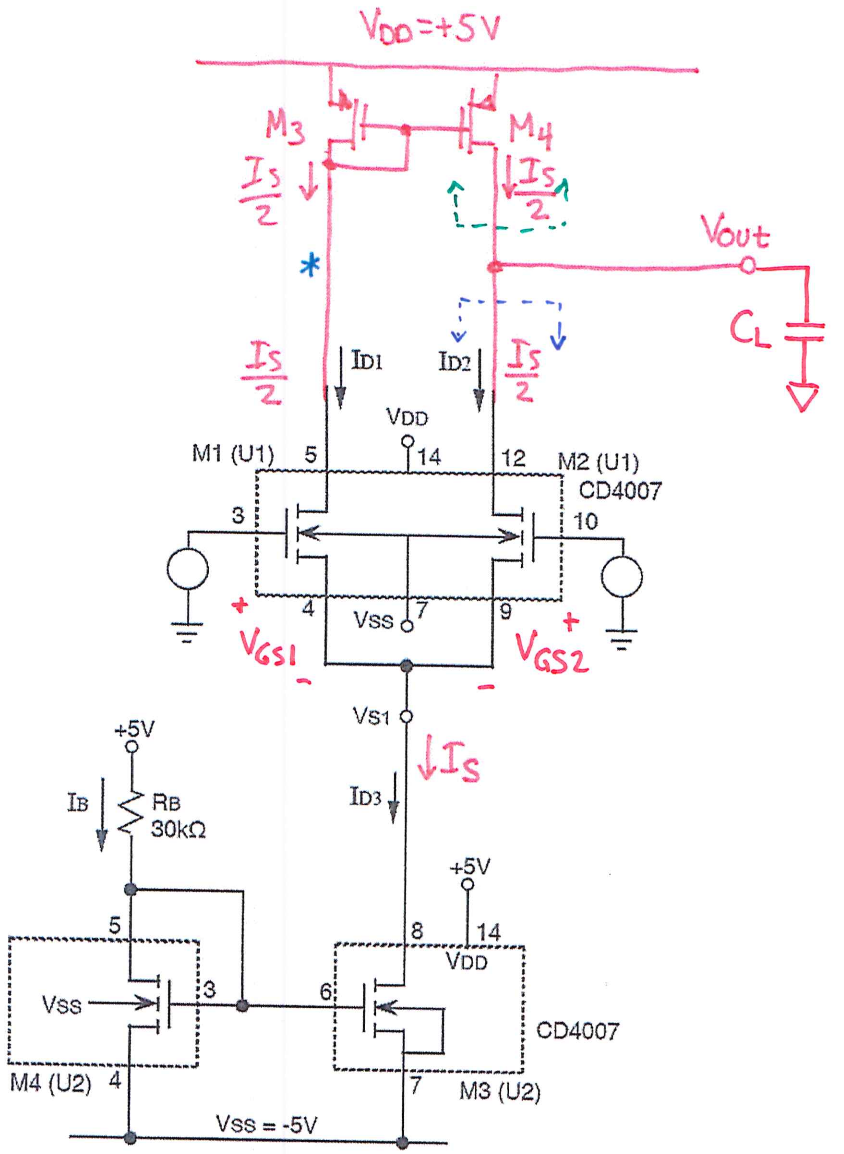
DIFFERENCE iout FLOWS IN Z_L AT LOAD

KCL: $\cancel{\frac{I_S}{2}} + g_m \frac{V_{id}}{2} = \cancel{\frac{I_S}{2}} - g_m \frac{V_{id}}{2} + i_{out}$

$g_m V_{id} = i_{out}$ } "RECOVER" "OTHER" HALF

$V_{out} = i_{out} Z_L = g_m V_{id} Z_L \Rightarrow \boxed{\frac{V_{out}}{V_{id}} = g_m Z_L}$

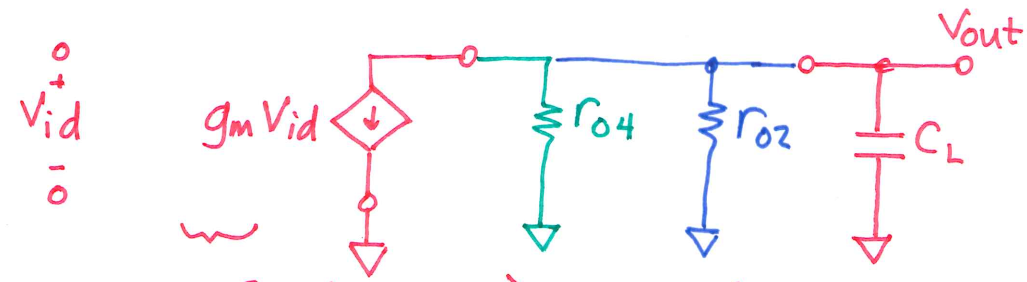
Differential Pair with Mirror Load: Small Signal Model



TRANSCONDUCTANCE OF DIFF PAIR

LOOKING INTO DRAIN OF M4

LOOKING INTO DRAIN OF M2



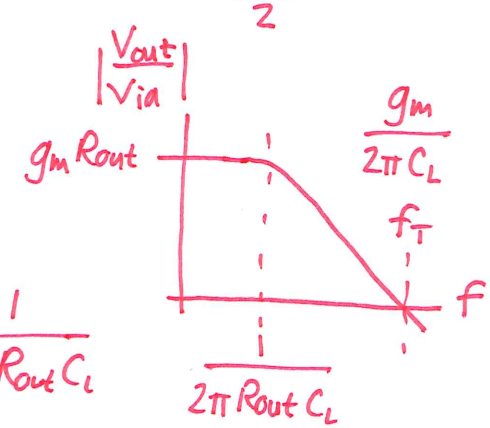
$$g_m = \frac{2 I_D}{(V_{GS1} - V_{TH})} \qquad I_D \rightarrow I_S/2$$

$$R_{out} = r_{o2} \| r_{o4} = \frac{1}{(\lambda_2 + \lambda_4)\underbrace{I_D}_{\frac{I_S}{2}}}$$

TRANSFER FUNCTION

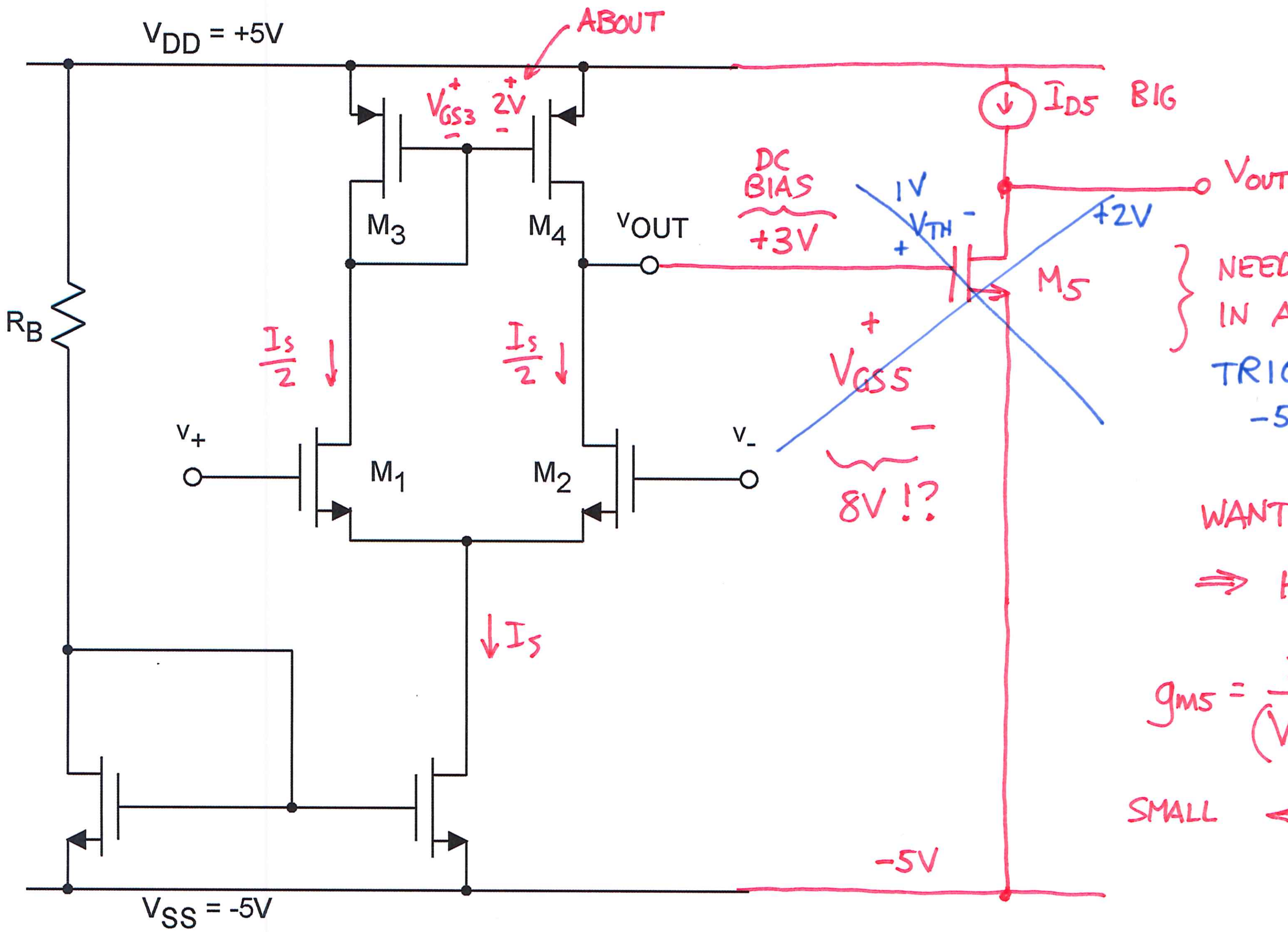
$$\frac{V_{out}}{V_{id}} = \frac{g_m R_{out}}{1 + s R_{out} C_L}$$

$$f_{3dB} = \frac{1}{2\pi R_{out} C_L}$$



2nd Stage Common Source Amplifier: NMOS or PMOS?

COMMON SOURCE WITH ACTIVE LOAD



NEED THIS TO BE IN ACTIVE REGION

TRIODE CRASH!
-5V TO +2V

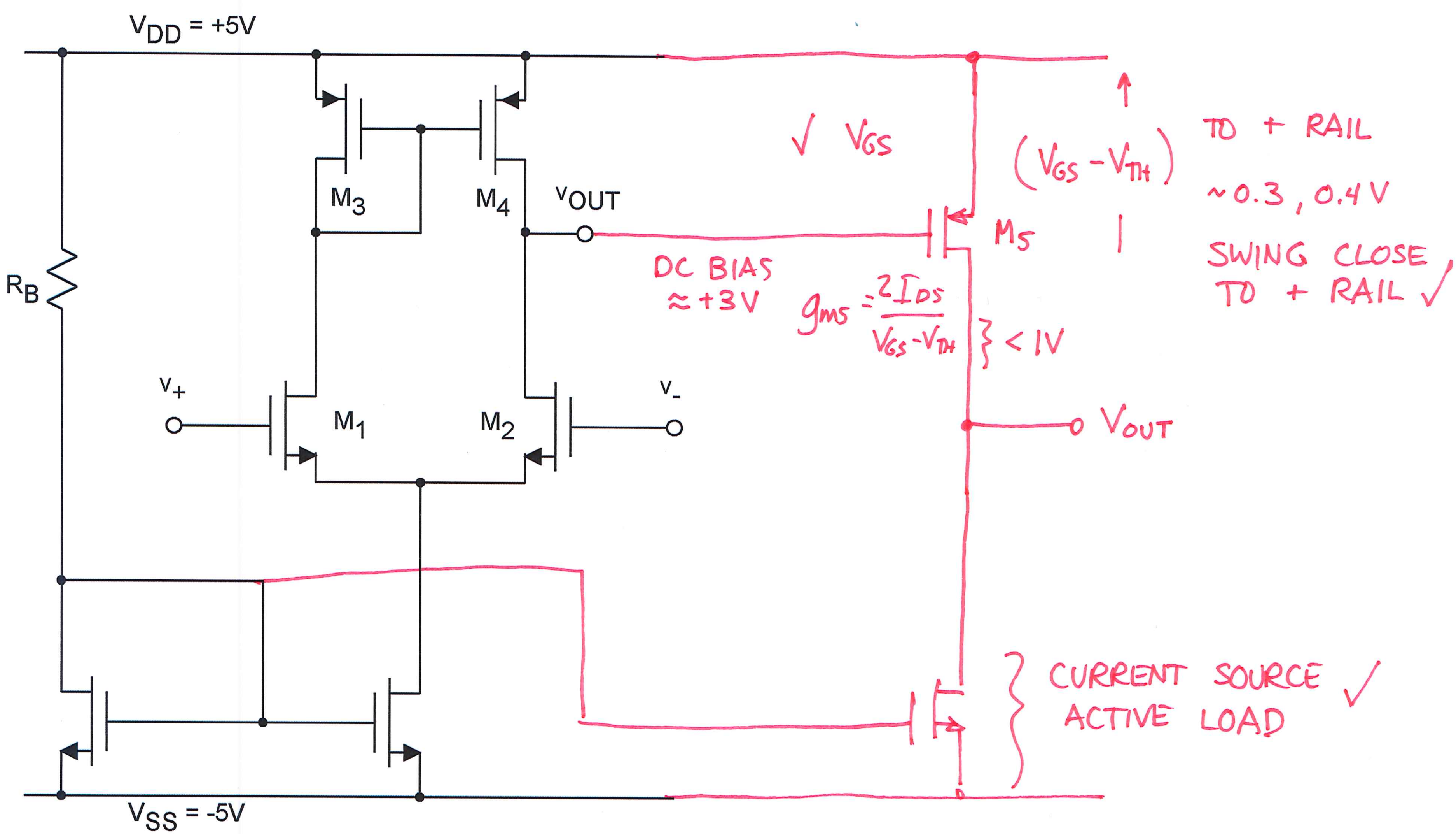
WANT HIGH GAIN:

⇒ HIGH g_{m5}

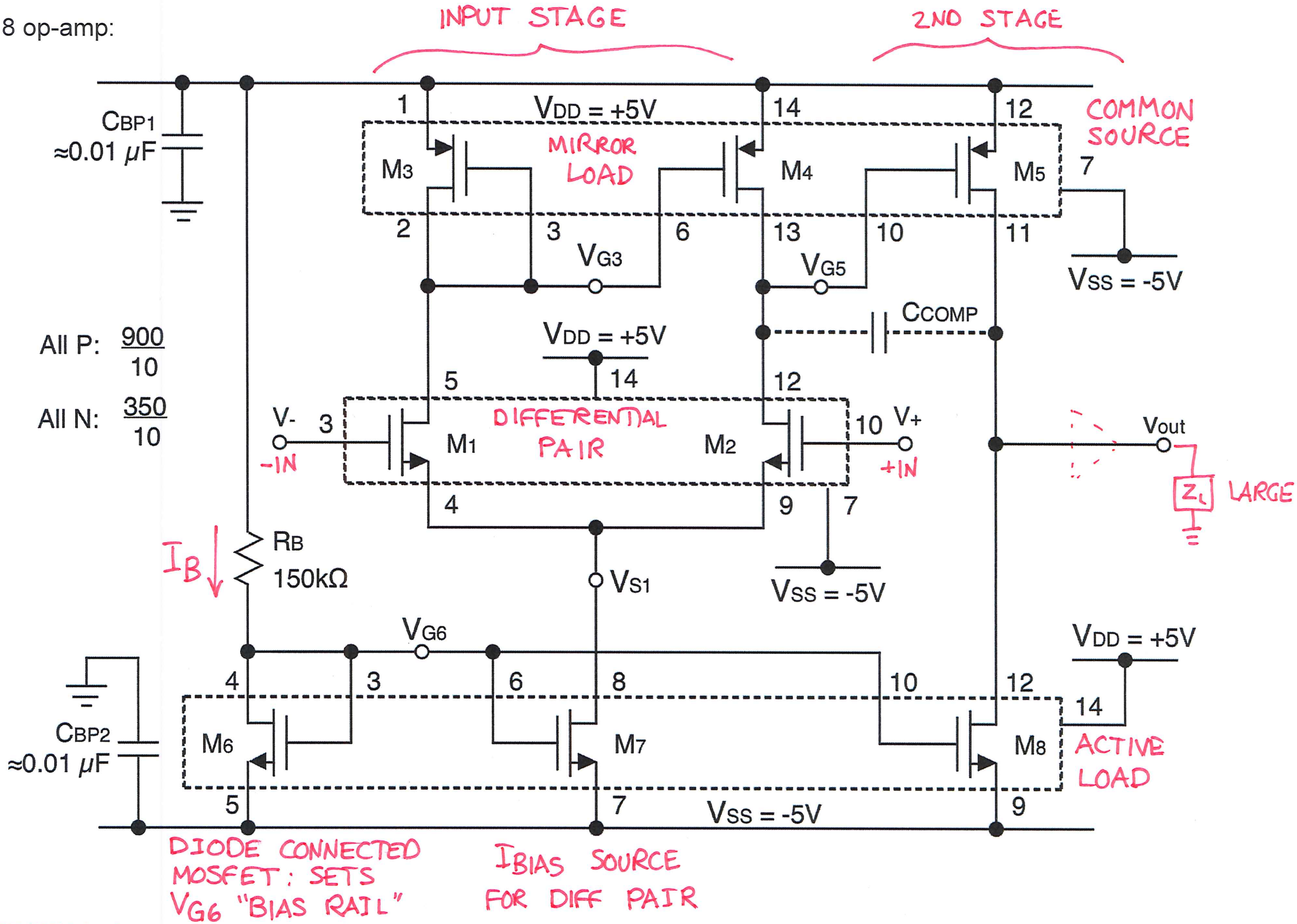
$$g_{m5} = \frac{2 I_{D5}}{\underbrace{(V_{GS5} - V_{TH5})}_{8V\ \text{BIG}}}$$

SMALL

2nd Stage Common Source Amplifier: NMOS or PMOS? FOR COMMON SOURCE?



Lab 8 op-amp:

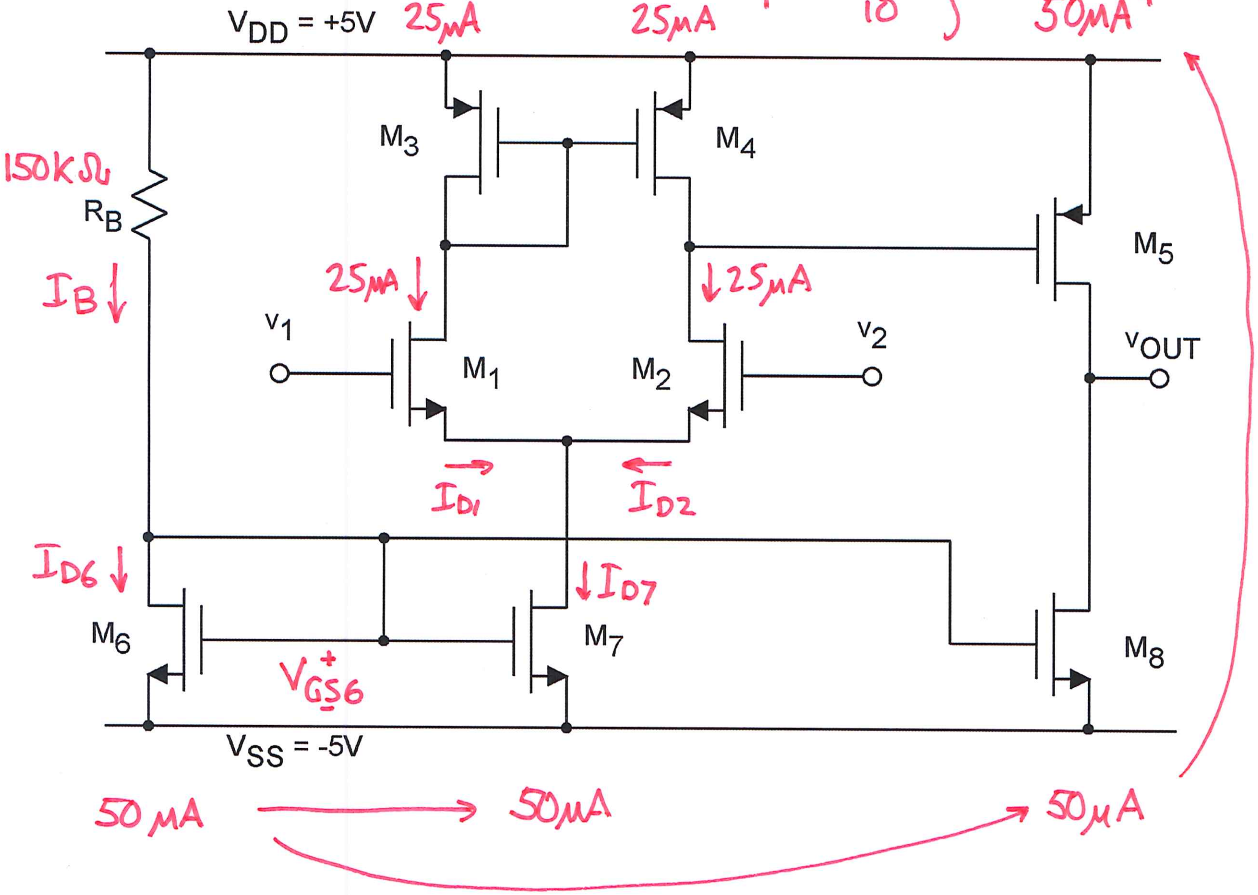


INPUT STAGE
2ND STAGE
COMMON SOURCE
CBP1 ≈0.01 μF
VDD = +5V
MIRROR LOAD
M3
M4
M5
VG3
VG5
VSS = -5V
CCOMP
All P: 900/10
All N: 350/10
VDD = +5V
DIFFERENTIAL PAIR
M1
M2
V- -IN
V+ +IN
Vout
ZL LARGE
IB
RB 150kΩ
VS1
VSS = -5V
VG6
VDD = +5V
CBP2 ≈0.01 μF
M6
M7
M8
ACTIVE LOAD
VSS = -5V
DIODE CONNECTED MOSFET: SETS VG6 "BIAS RAIL"
IBIAS SOURCE FOR DIFF PAIR

ALL N $\frac{350}{10}$
P $\frac{900}{10}$ } CD4007

Op-amp DC Operating Point Analysis

Parameter	N-channel	P-channel	Units
V_{TH}	+1.90	-1.60	V
μC_{ox}	2.6E-5	9.1E-6	A/V^2
λ (L=10μm)	0.05	0.028	V^{-1}



STRATEGY: FOLLOW CURRENT AROUND

START: I_B IN BIAS FOR MIRROR

OHM'S LAW FOR R_B:

$$+5V - I_B R_B - V_{GS6} = -5V$$

$$I_B = \frac{10V - V_{GS6}}{R_B} \; \} \text{ SQ LAW } I_{D6} = I_B$$

SOLVE QUADRATIC ☹

NEED ±20%

GUESS FOR V_{GS6} = 2.5V ?!

TRY $\frac{10V - 2.5V}{150k\Omega} = 50\ \mu A$

CHECK IN SQUARE LAW:

$$50\mu A = \frac{2.6E\text{-}5}{2}\,\frac{350}{10}\,(V_{GS6} - 1.9V)^2$$

$\Rightarrow V_{GS6} = 2.23V$

REVISE I_B

$$\frac{10V - 2.23V}{150k\Omega} = 51\ \mu A$$

$$\underbrace{I_{D1} + I_{D2}}_{\text{EQUAL}} = I_{D7} = 50\mu A$$

V_{GS} VALUES FROM SQ LAW

$I_{D6} \rightarrow I_{D7}, I_{D8} \rightarrow I_{D5} = 50\mu A$ (WITH NEGATIVE FEEDBACK)

$\downarrow$

$I_{D1} = I_{D2} = 25\ \mu A$

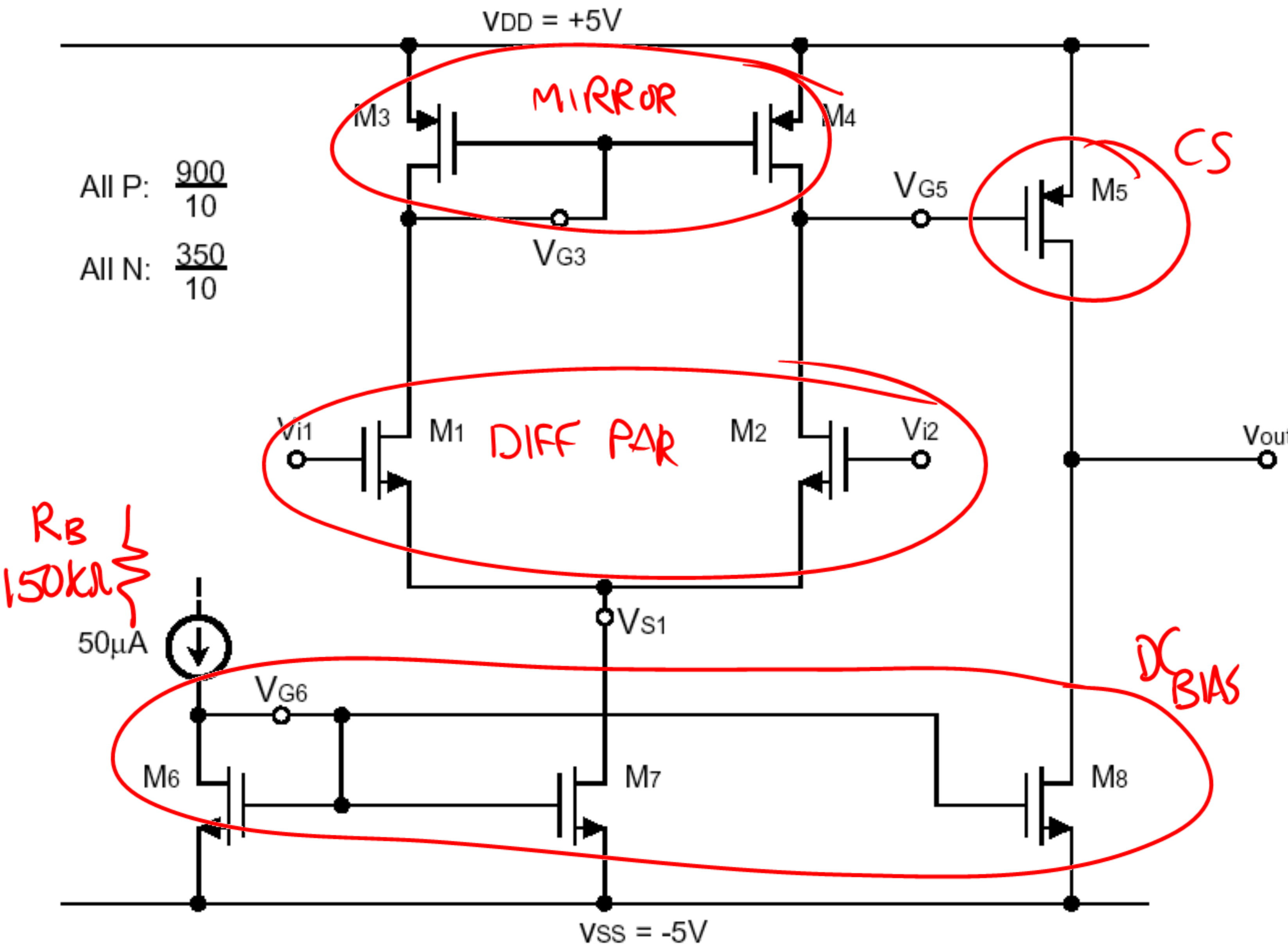
$\downarrow$

$\underset{I_{D3}}{M_3} = 25\mu A \rightarrow M_4 : I_{D4} = 25\mu A$

Labs 8 & 9 Review

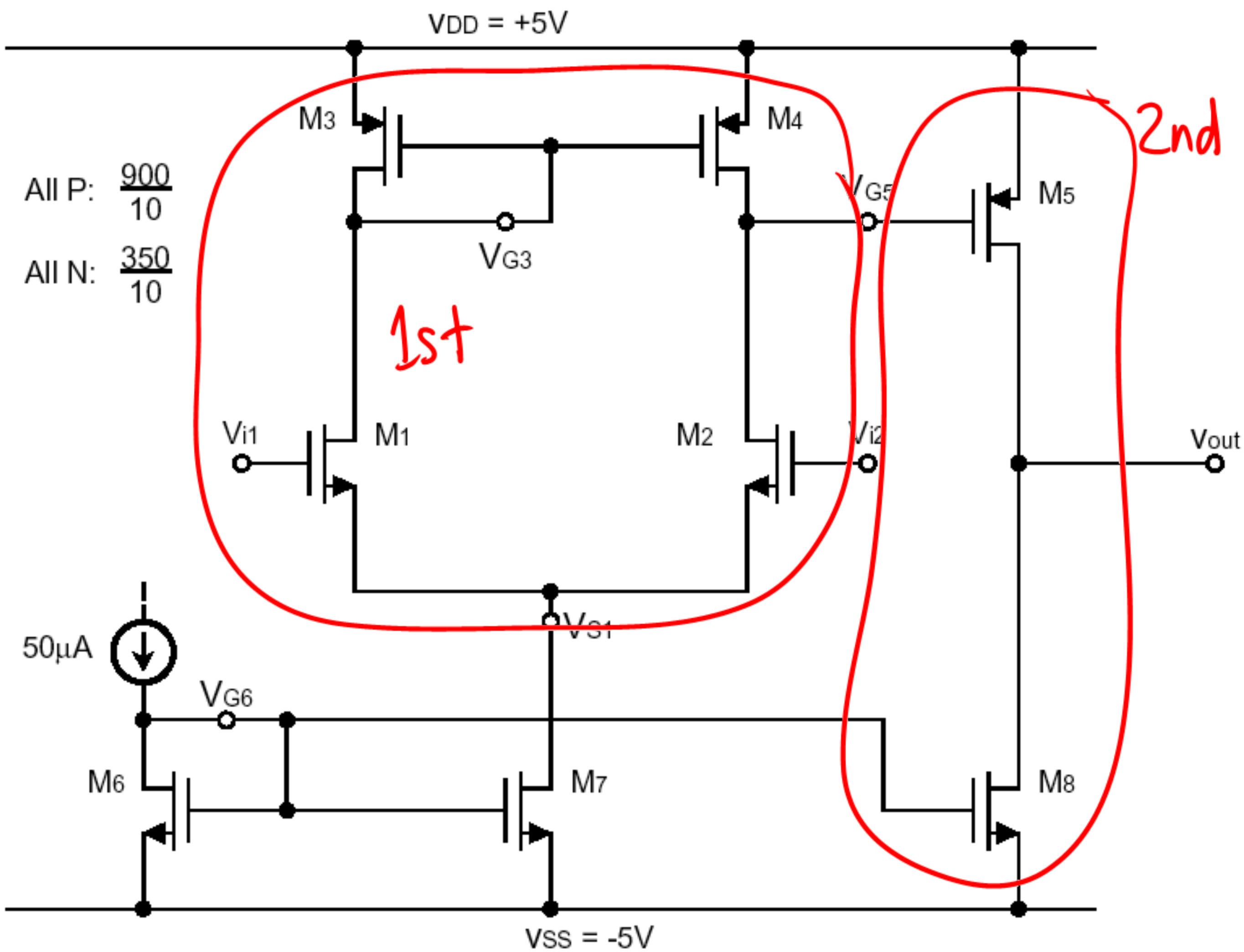
- **Operational Amplifier**
  - **Stability**
  - **Compensation**
  - **Miller Effect**
  - **Phase Margin**
  - **Unity Gain Frequency**
  - **Slew Rate Limiting**
- **Reading: Razavi ch. 9, 10**
  - **Lab 8, 9 op-amp is Fig. 10.34 in sec. 10.5.1**
  - **(see also Johns & Martin sec 5.2 pp. 232-242)**

Two-stage op-amp

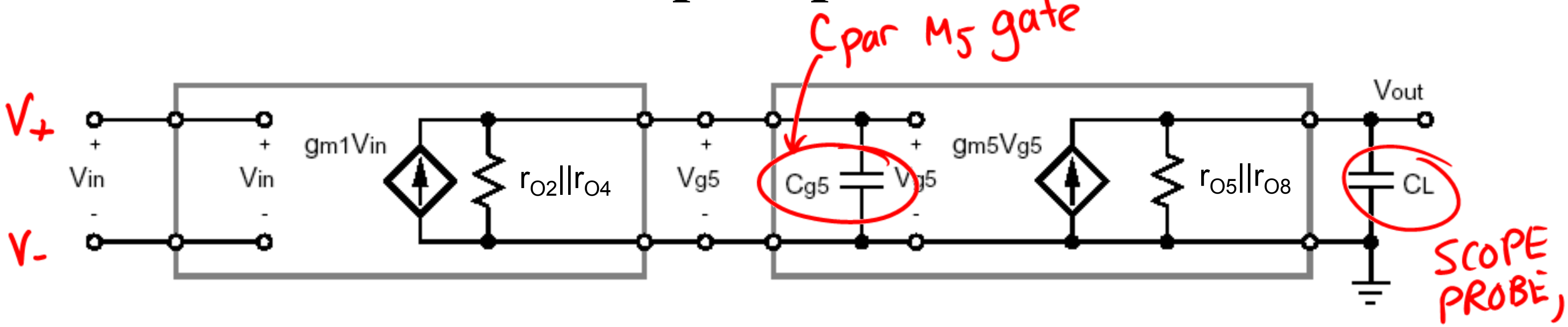


Analysis Strategy

- **Recognize sub-blocks**
- **Represent as cascade of simple stages**



Total op-amp model



Input differential pair **Common source stage** Cpar

$$g_{m1} = \frac{2I_{D1}}{(V_{GS1} - V_{TH1})}$$

$$g_{m2} = \frac{2I_{D5}}{(V_{G5} - V_{TH5})}$$

$$r_{o2}||r_{o4} = \frac{1}{(\lambda_2 + \lambda_1) I_{D1}}$$

$$r_{o5}||r_{o8} = \frac{1}{(\lambda_5 + \lambda_8) I_{D5}}$$

DC operating point

	$I_D[\mu A]$	$V_{GS}-V_{TH}$
M1	25	0.235
M2	25	0.235
M3	25	0.247
M4	25	0.247
M5	50	0.350
M6	50	0.332
M7	50	0.332
M8	50	0.332

ID ANALYSIS

SQ LAW

Small signal parameters

	$I_D[\mu A]$	$V_{GS}-V_{TH}$	$g_m[\mu A/V]$	r_O
M1	25	0.235	208	
M2	25	0.235		800kΩ
M3	25	0.247		
M4	25	0.247		1.43MΩ
M5	50	0.350	285	715kΩ
M6	50	0.332		
M7	50	0.332		
M8	50	0.332		400kΩ

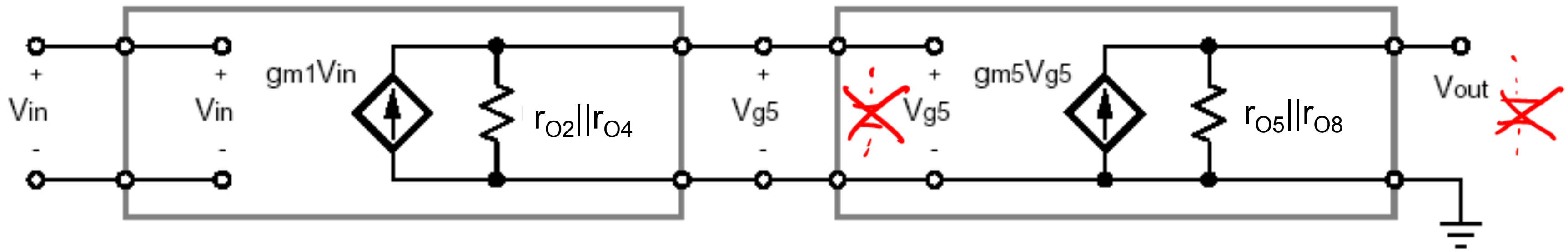
1st

2nd

Note: $\lambda_n = 0.050\ V^{-1}$; $\lambda_p = 0.028\ V^{-1}$

Total op-amp model: Low frequency gain

CAP OPEN



Input differential pair

$$a_{v1} = g_{m1}\left(r_{O2} \| r_{O4}\right)$$

$$a_{v1} = \left(208\mu A/V\right)\left(800k\Omega \| 1.43M\Omega\right)$$

$$a_{v1} = 106$$

Common source stage

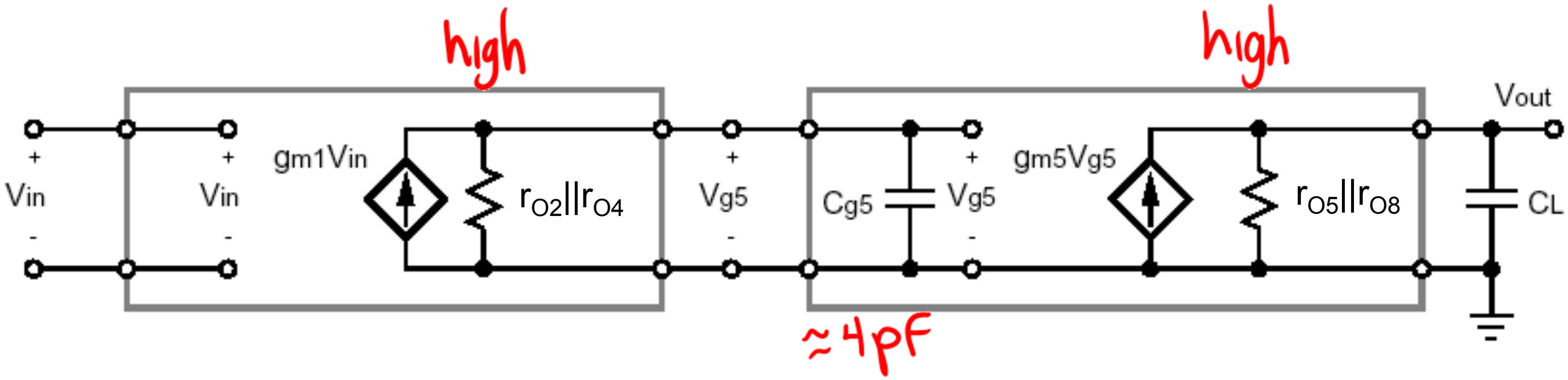
$$a_{v2} = g_{m2}\left(r_{O5} \| r_{O8}\right)$$

$$a_{v2} = \left(285\mu A/V\right)\left(400k\Omega \| 715k\Omega\right)$$

$$a_{v2} = 73$$

$$\frac{V_{out}}{V_{in}} \simeq 7000$$

Total op-amp model with capacitances



Gate of M5 **Load: scope probe ≈10pF**

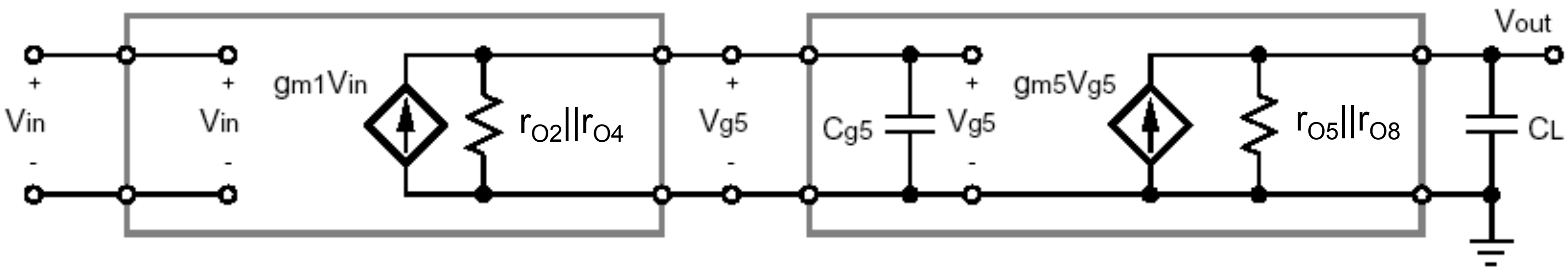
$$C_g = (900\,\mu m)(10\,\mu m)\left(4.17E-4\frac{F}{m^2}\right)$$

$$C_g = 3.74\,pF$$

EACH CAP: 1 POLE TO TRANSFER FUNCTION $\frac{V_{out}}{V_{in}}(f)$

ONLY 2 $\underbrace{\text{HIGH IMPEDANCE}}_{\text{DOMINANT POLE}}$ NODES: V_{g5}, V_{out}

Total op-amp model with capacitances



First stage pole

$$f_{p1} = \frac{1}{2\pi\left(r_{O2} \| r_{O4}\right)C_{g5}}$$

$$f_{p1} = \frac{1}{2\pi\left(800k\Omega \| 1.43M\Omega\right)\left(3.74\,pF\right)}$$

$$f_{p1} = 82kHz$$

Second stage pole

$$f_{p1} = \frac{1}{2\pi\left(r_{O5} \| r_{O8}\right)C_{L}}$$

$$f_{p1} = \frac{1}{2\pi\left(400k\Omega \| 715k\Omega\right)\left(10\,pF\right)}$$

$$f_{p1} = 61kHz$$

Open loop transfer function

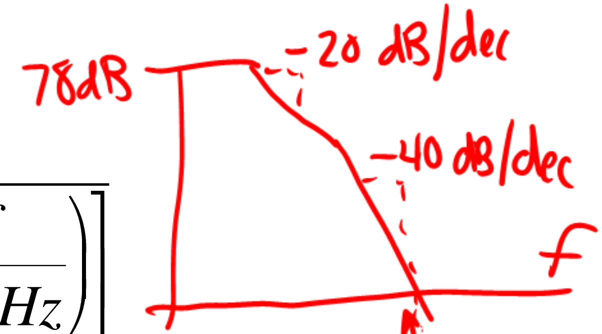
- **Product of individual stage transfer functions**

$$A(j\omega) = \frac{\overbrace{g_{m1}\left(r_{O2}\|r_{O4}\right)}^{106}\overbrace{g_{m5}\left(r_{O5}\|r_{O8}\right)}^{73}}{\left[1 + j\omega\left(r_{O2}\|r_{O4}\right)C_{g5}\right]\left[1 + j\omega\left(r_{O5}\|r_{O8}\right)C_L\right]}$$

1st 2nd

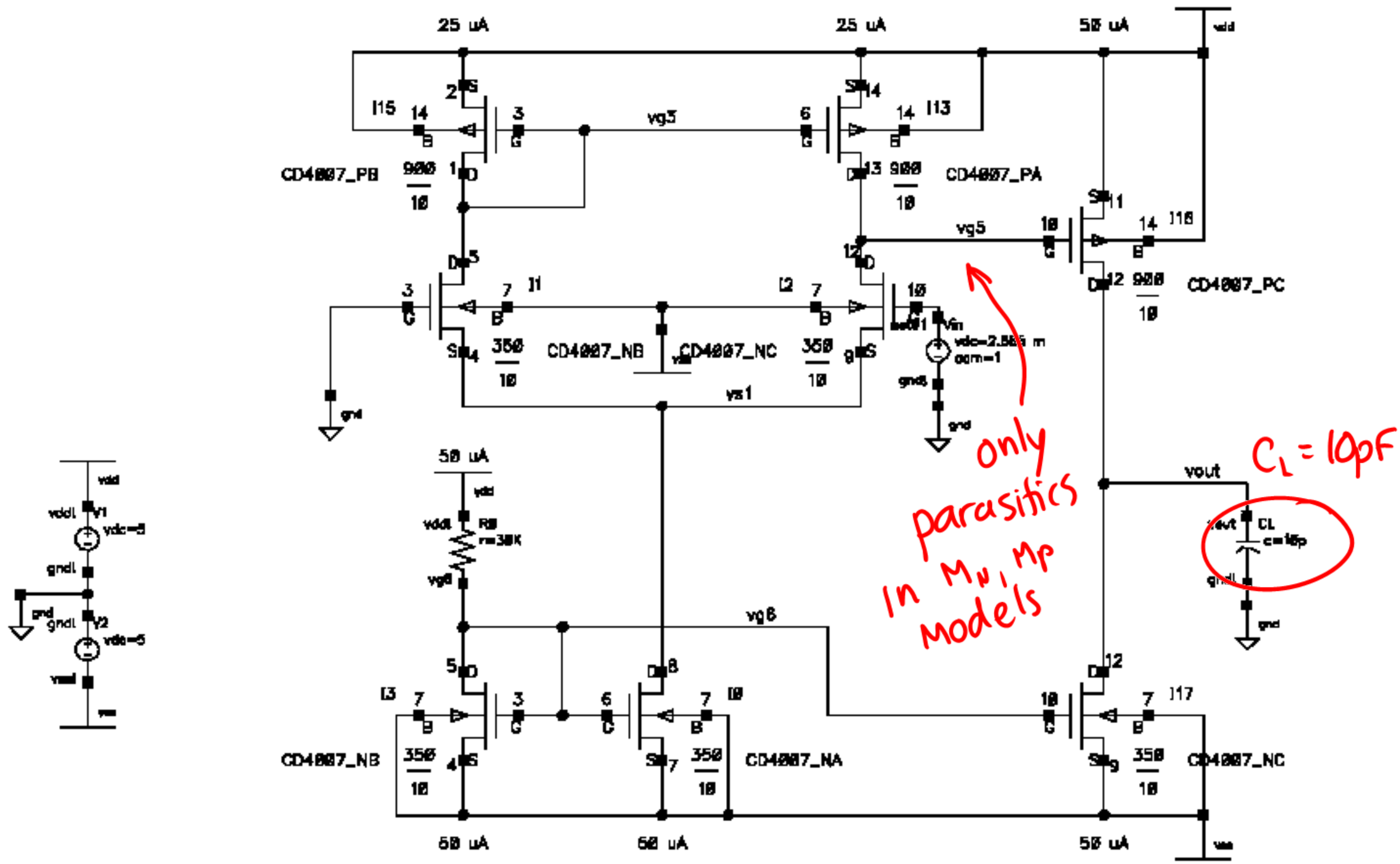
- **Numerically (using ω = 2πf)**

$$A(j\omega) = \frac{7738}{\left[1 + j\left(\frac{f}{82kHz}\right)\right]\left[1 + j\left(\frac{f}{61kHz}\right)\right]}$$

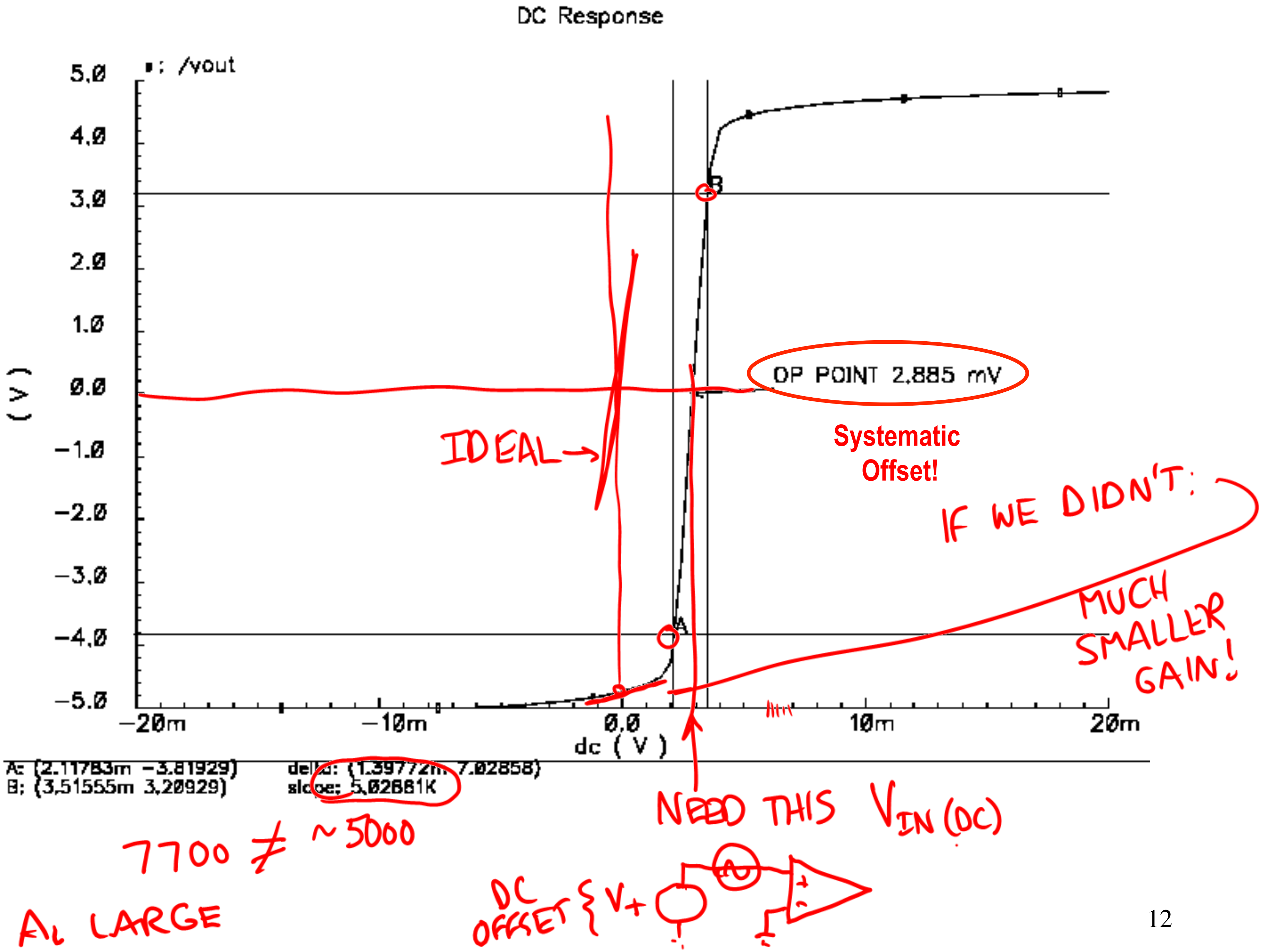


- **Check Bode plot simulation; predicts:**
  - **DC gain = 20log(7738) = +78dB**
  - **Unity gain frequency ~ 6.2 MHz** (SET |A(ω)|=1)

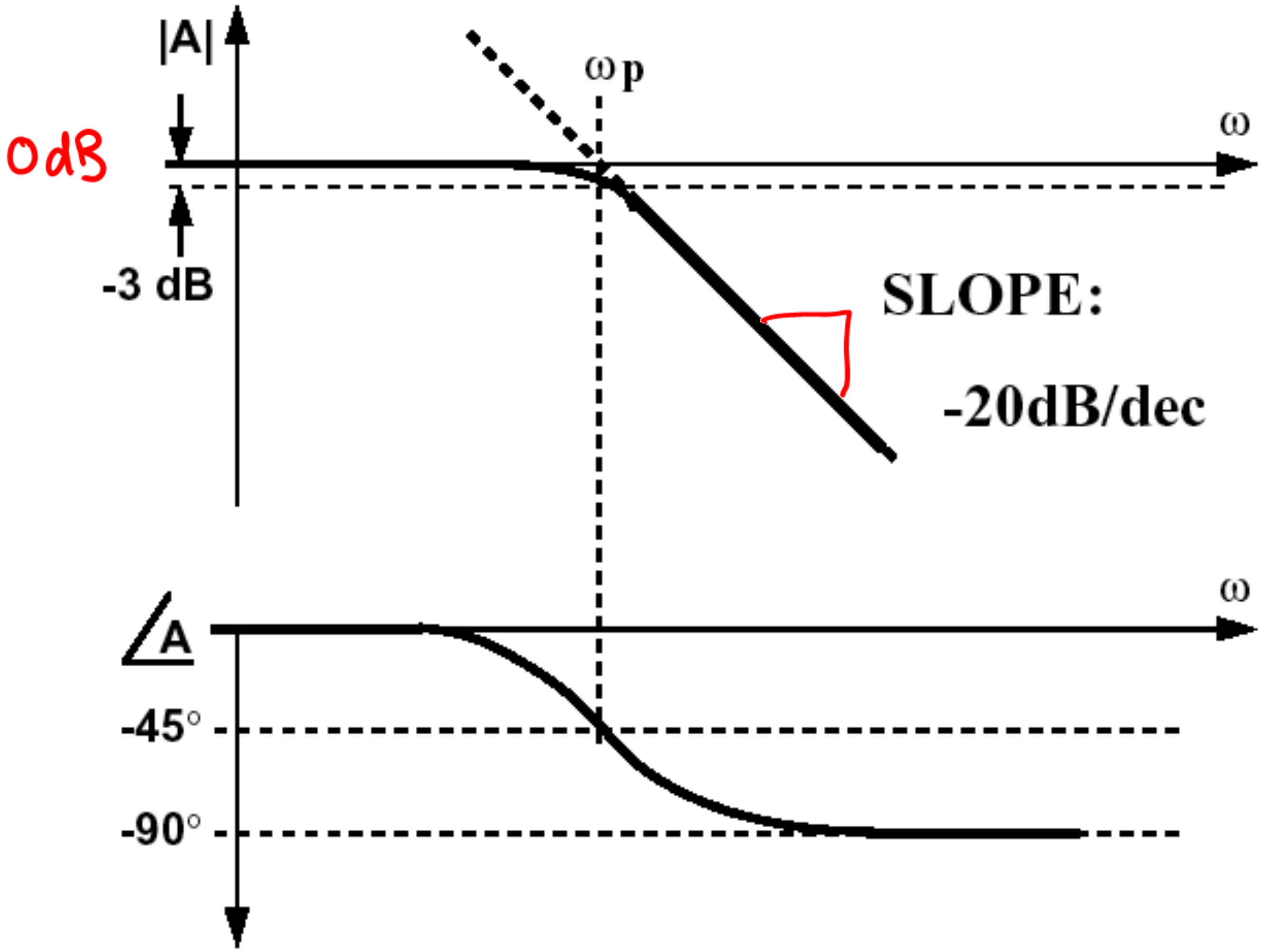
Two-stage op-amp: Simulation Schematic



DC Operating Point Simulation



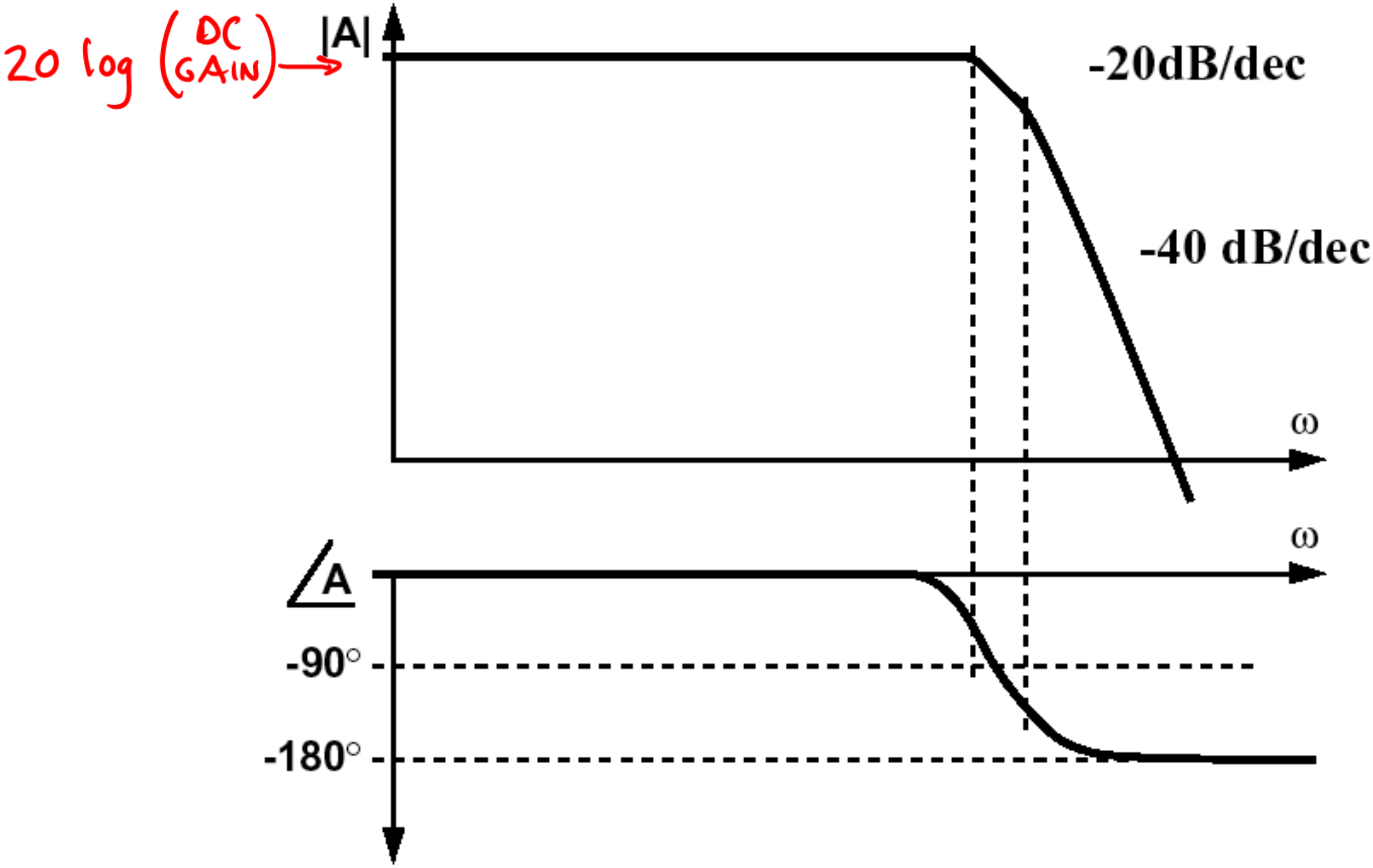
Bode plot EACH POLE TERM



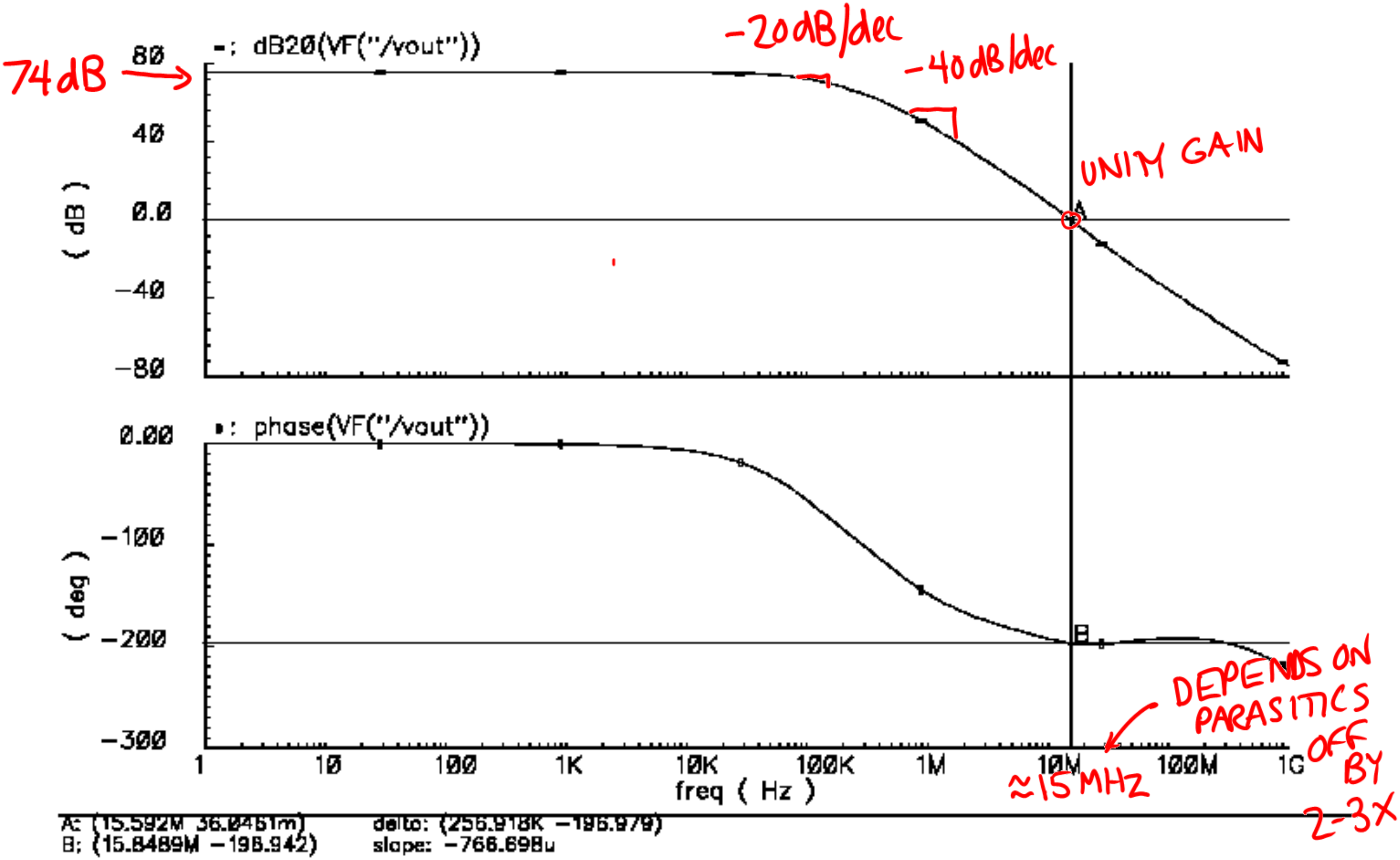
- **Magnitude, phase on log scales**
- **Pole: Root of denominator polynomial**

Open loop Bode plot

- **Product of terms : Sum on log-log plot**

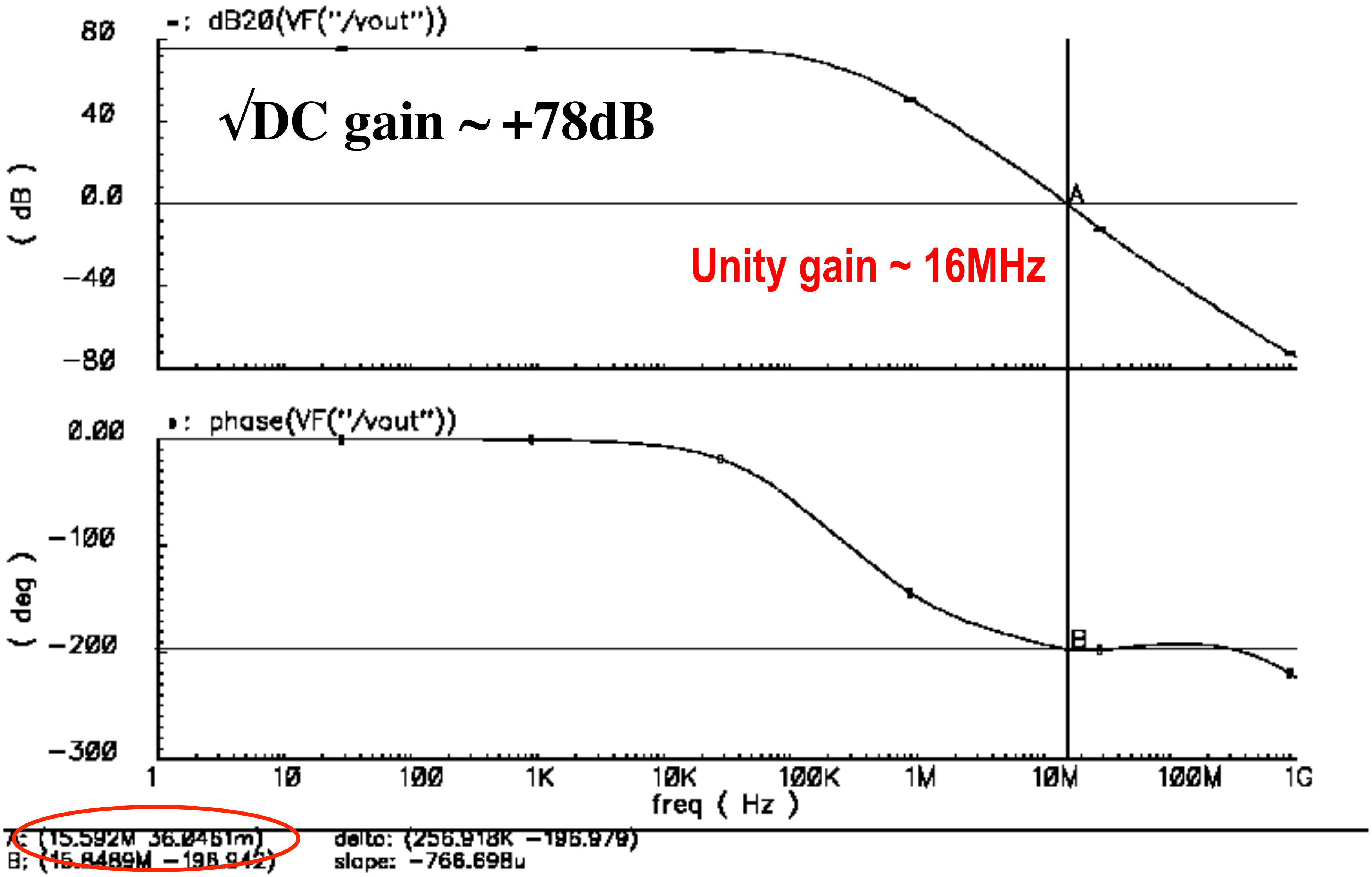


Open Loop Bode Plot Simulation

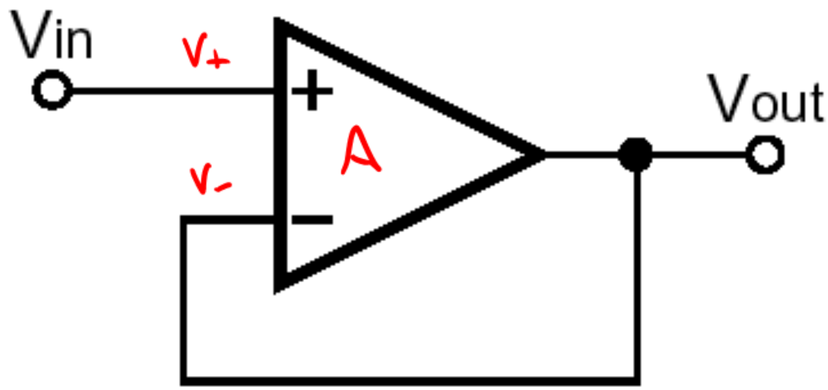


Note: AC source at input also needs DC component to account for systematic offset!

Check Open Loop Bode Plot Simulation



Stability example: Closed loop follower



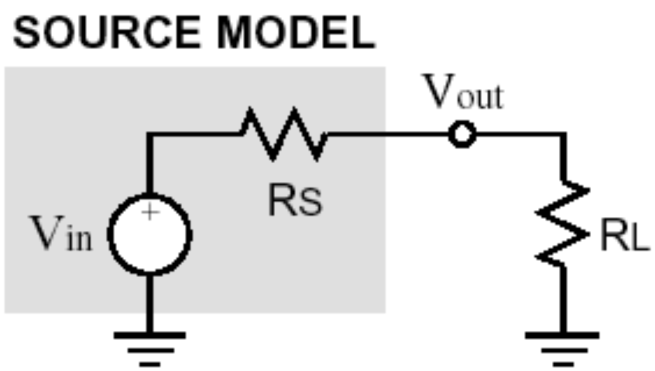
- **Negative feedback:**
  Output connected to inverting input
- **Gain should be ~ 1**

$$v_{out} = A\left(v_{in} - v_{out}\right)$$

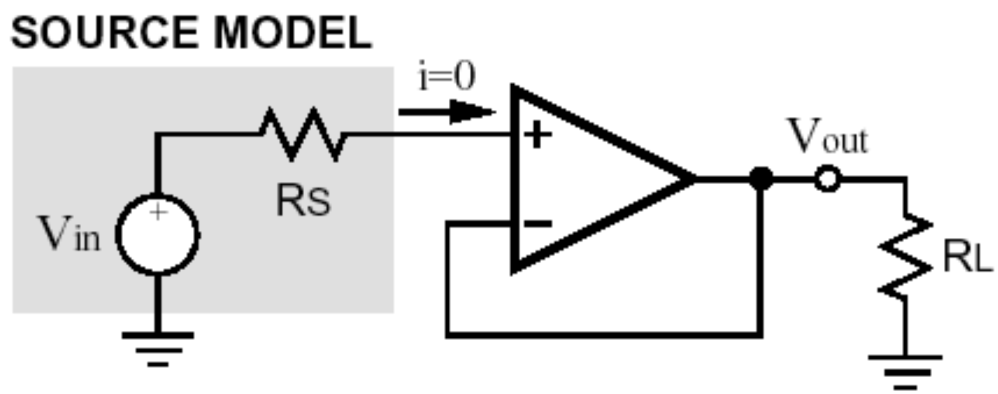
$$v_{out}\left(A+1\right) = Av_{in}$$

$$v_{out} = \underbrace{\left(\frac{A}{A+1}\right)}_{\approx 1\, as\, A >> 1} v_{in}$$

Unity gain: Why bother?



$$v_{out} = \left(\frac{R_L}{R_L + R_S}\right) v_{in}$$



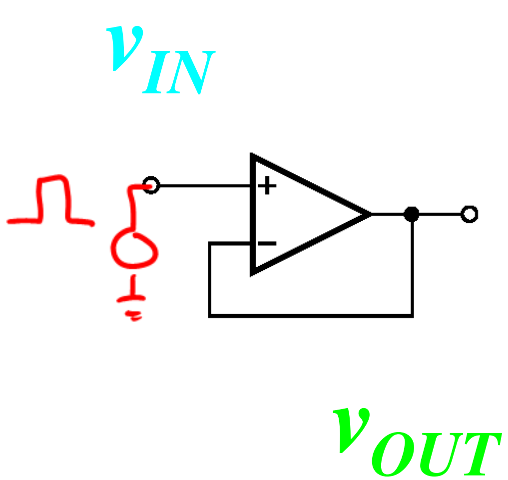
$$v_{out} = v_{in}$$

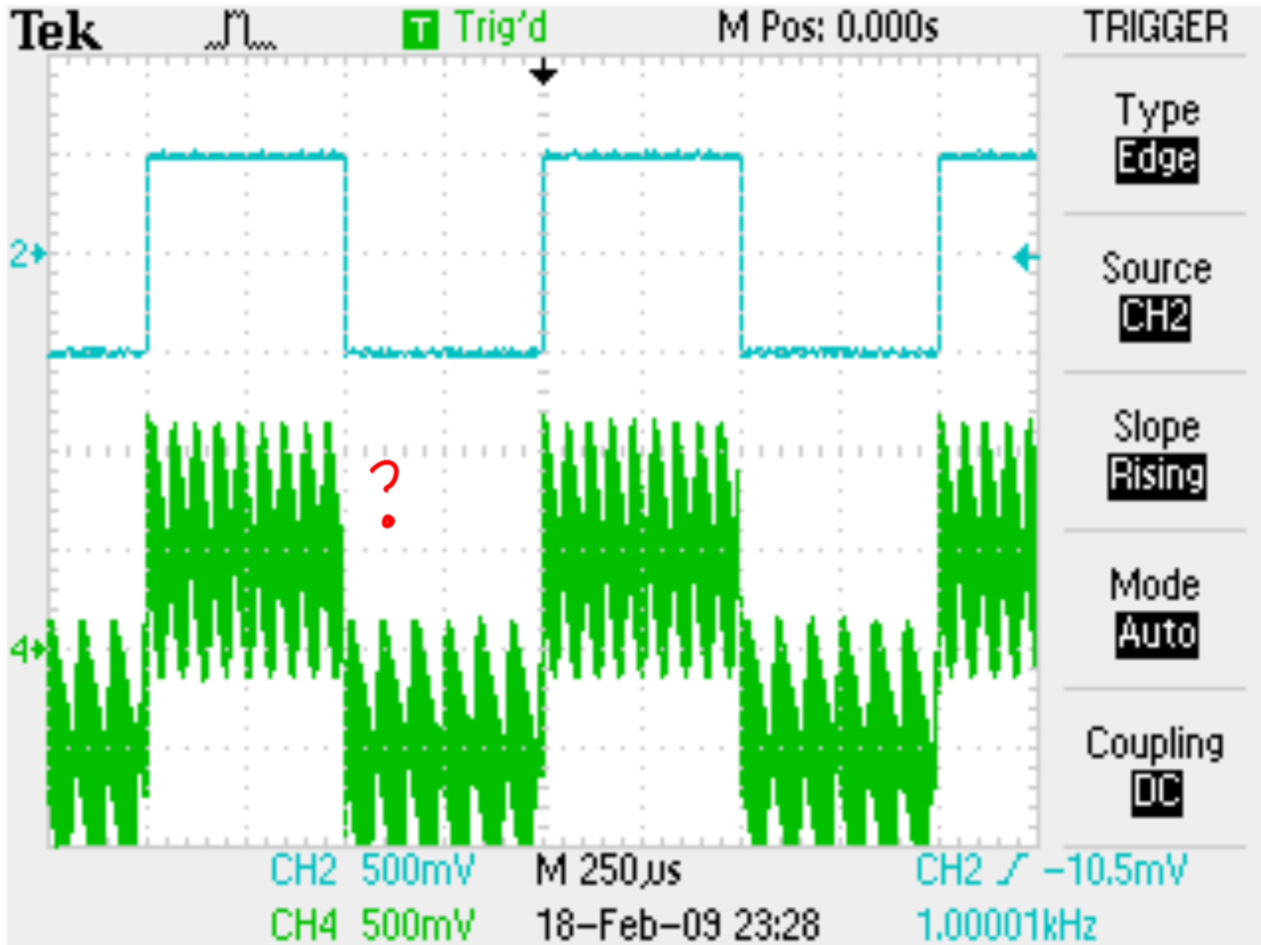
- **No buffer: Voltage divider**
- **Signal reduced due to voltage drop across R_S**

- **With buffer: No current required from source**

Lab 9 Problem: Instability

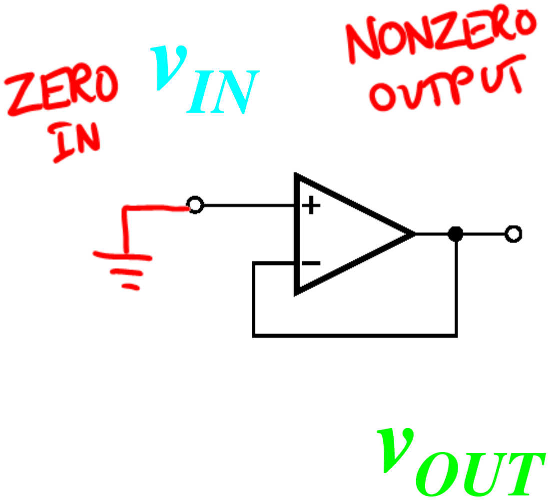
- **Oscillation superimposed on desired output!?!**

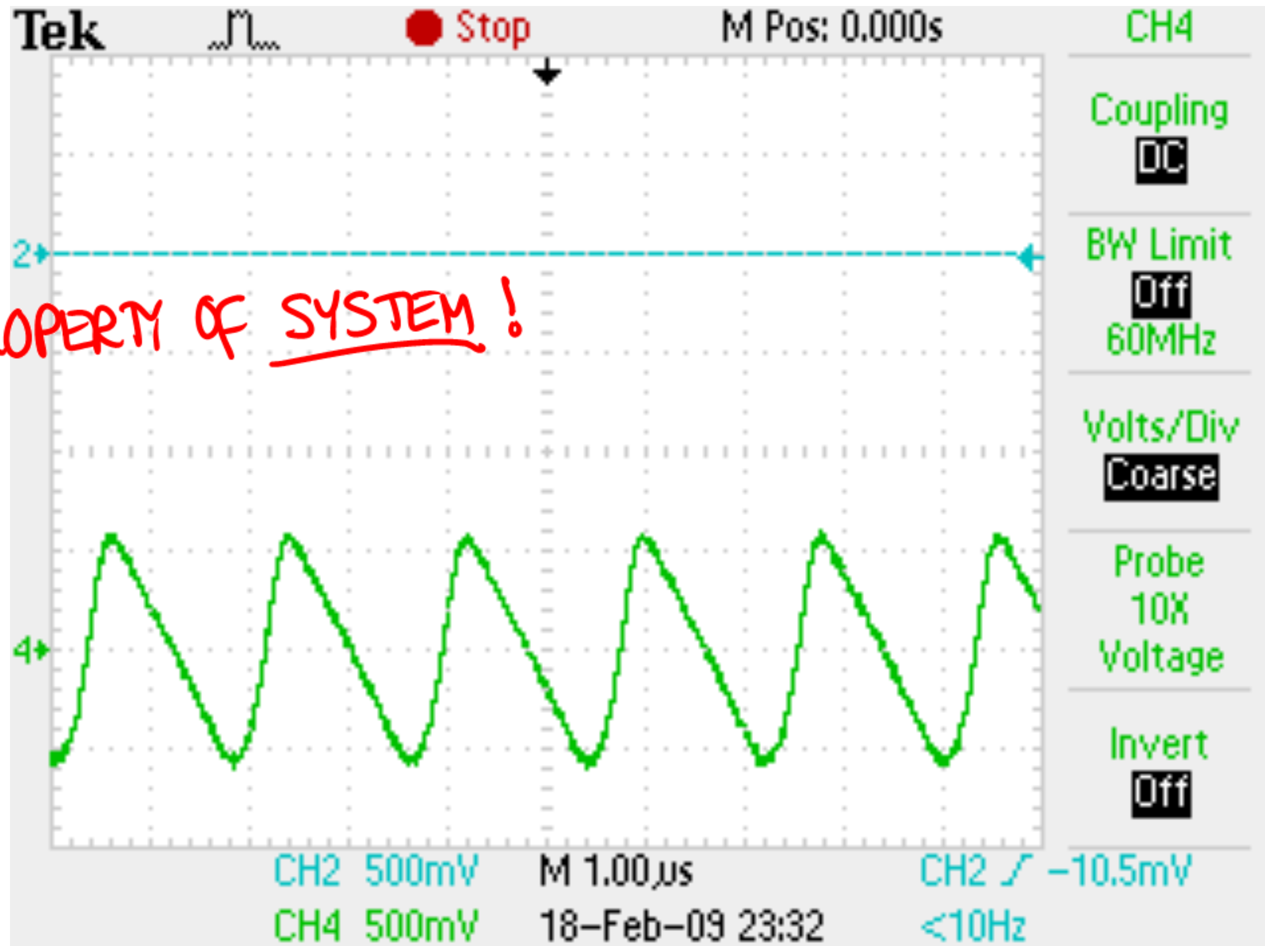




Lab 9 Problem: Instability

- **Ground v_{in}: Output for zero input?!?**
- **Why? Need...**





PROPERTY OF SYSTEM!

FOR THIS WAVEFORM

$$\frac{V_{out}}{V_{in}} = \infty$$

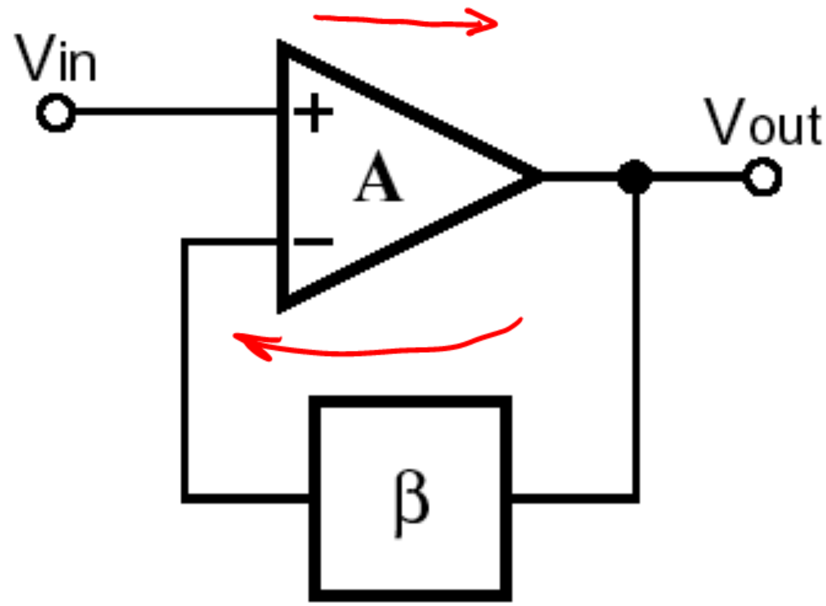
Controls: ECE3012 in 20 minutes

- **General framework**

  A: Forward Gain

  β: Feedback Factor

  fraction of output fed back to input



Example: Op-amp, Noninverting Gain

A: Forward Gain

Op-amp open loop gain

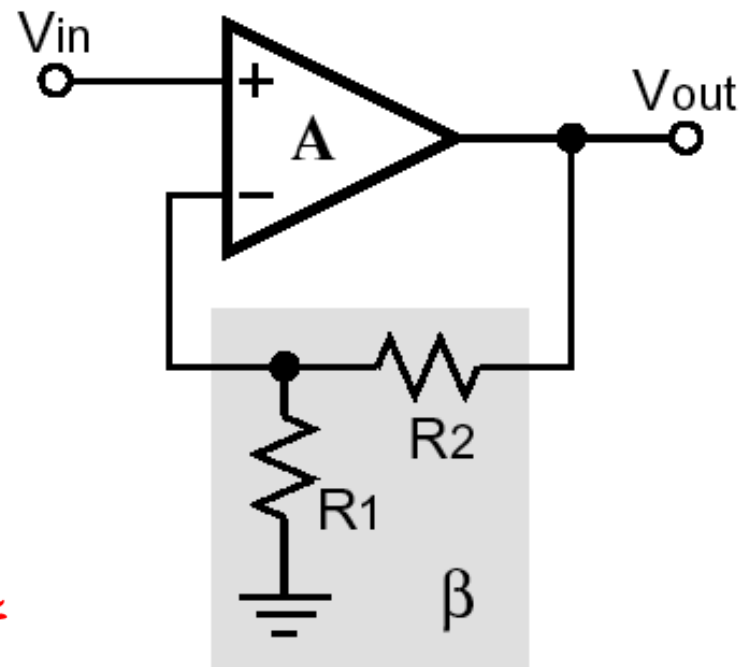
$V_{out}=A(V_+-V_-)$

Transfer function $A(j\omega)$

β: Feedback Factor

$$\beta = \frac{R_1}{R_1 + R_2}$$

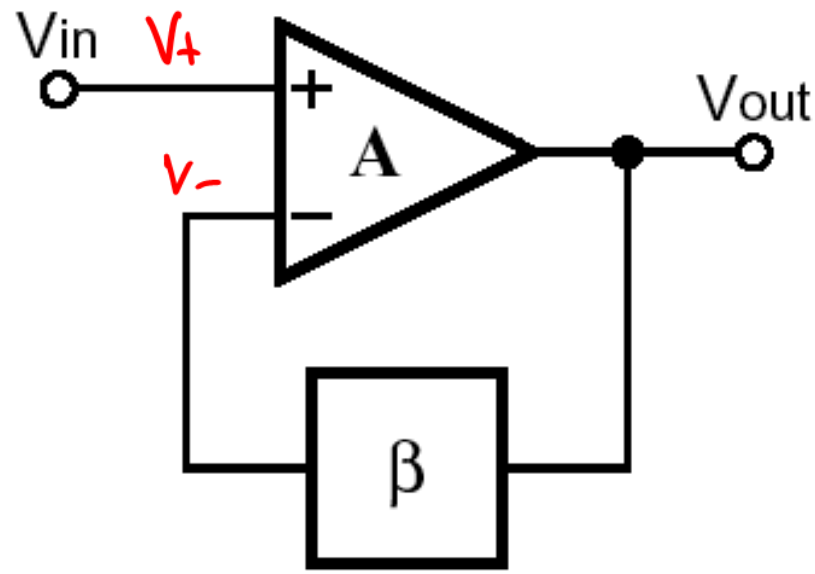
VOLTAGE DIVIDER



Closed Loop Gain

- **Output**

$$v_{out} = A\big(\underbrace{v_{in} - \beta v_{out}}_{v_+ - v_-}\big)$$



- **Solve for $\mathbf{v_{out}/v_{in}}$**

$$v_{out} = Av_{in} - A\beta v_{out}$$

$$(1 + A\beta)v_{out} = Av_{in}$$

$$\boxed{\frac{v_{out}}{v_{in}} = \frac{A}{1 + A\beta}}$$

Op-amp with negative feedback

- **If Aβ >> 1**

$$\frac{v_{out}}{v_{in}} = \frac{A}{\cancel{1} + A\beta} \approx \frac{\cancel{A}}{\cancel{A}\beta} \quad \rightarrow \quad \frac{v_{out}}{v_{in}} \approx \frac{1}{\beta}$$

- **Closed loop gain determined only by β**
- **Advantage of negative feedback:**
  Open loop gain A can be ugly (nonlinear, poorly controlled) as long as it's large!

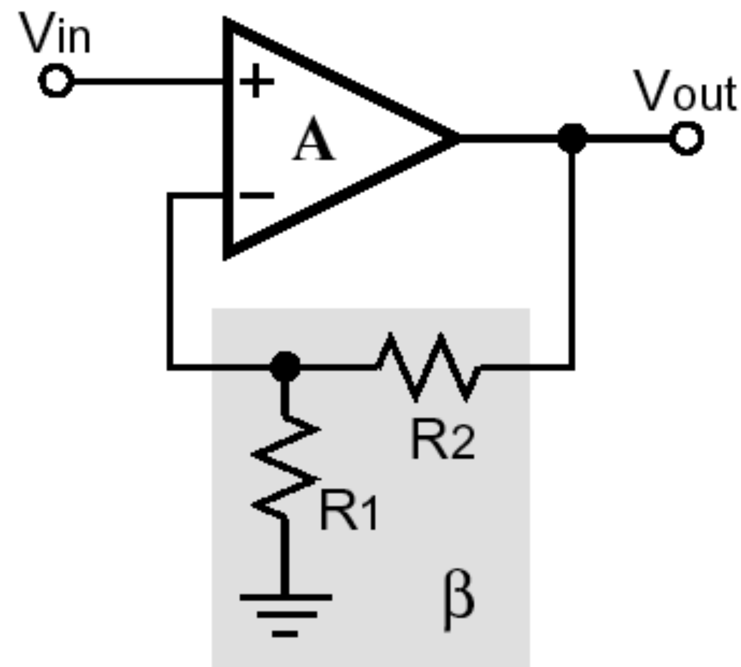
Example: Op-amp, Noninverting Gain

β: Feedback Factor

$$\beta = \frac{R_1}{R_1 + R_2}$$

Closed loop gain

$$\frac{v_{out}}{v_{in}} = \frac{R_1 + R_2}{R_1} = \frac{1}{\beta}$$



Reexamine closed loop transfer function

- **Output with no input: infinite gain**
- **Infinite when 1+Aβ = 0**
- **Condition for oscillation:**

$$\infty \quad \frac{v_{out}}{v_{in}} = \frac{A}{\underbrace{1 + A\beta}_{0}}$$

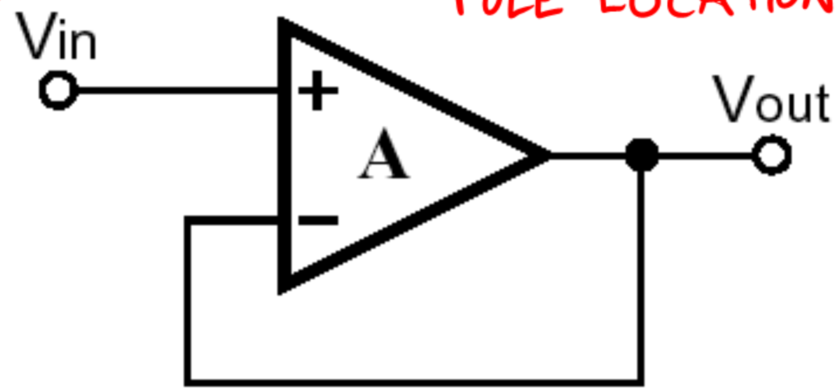
$$\mathbf{1+A\beta = 0}$$

- **In general A, β functions of ω**
- **If there's a frequency ω at which 1+Aβ = 0: Oscillation at that frequency!**

Example: follower

$$\beta = 1 \quad \rightarrow \quad \frac{v_{out}}{v_{in}} = \frac{A}{1 + A}$$

TOO HARD TO FIND EXACT POLE LOCATIONS!



$\beta = 1$

- **Use A(jω), solve for 1+A = 0**
- **No thanks!**

$$A(j\omega) = \frac{g_{m1}\left(r_{O2} \| r_{O4}\right) g_{m5}\left(r_{O5} \| r_{O8}\right)}{\left[1 + j\omega\left(r_{O2} \| r_{O4}\right) C_{g5}\right]\left[1 + j\omega\left(r_{O5} \| r_{O8}\right) C_L\right]}$$

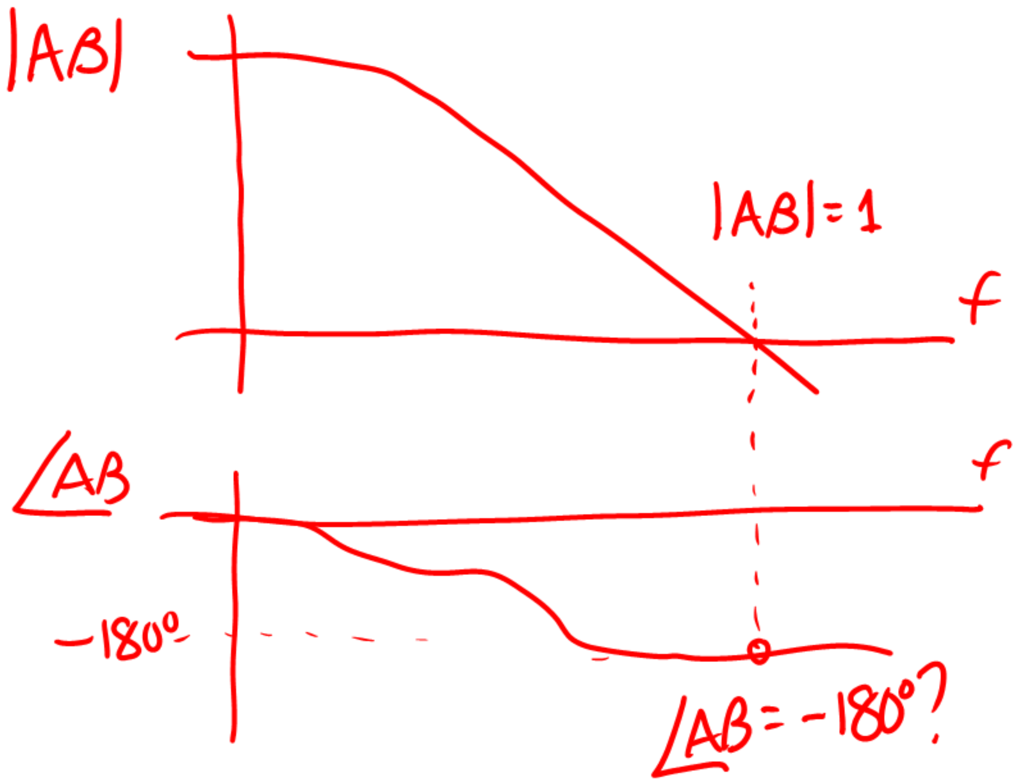
Reexamine condition for oscillation

DEN $\{ \quad 1+A\beta = 0 \rightarrow A\beta = -1$

Magnitude and phase condition:

$$\boxed{|A\beta| = 1 \;\; \text{AND} \;\; \angle A\beta = -180°}$$

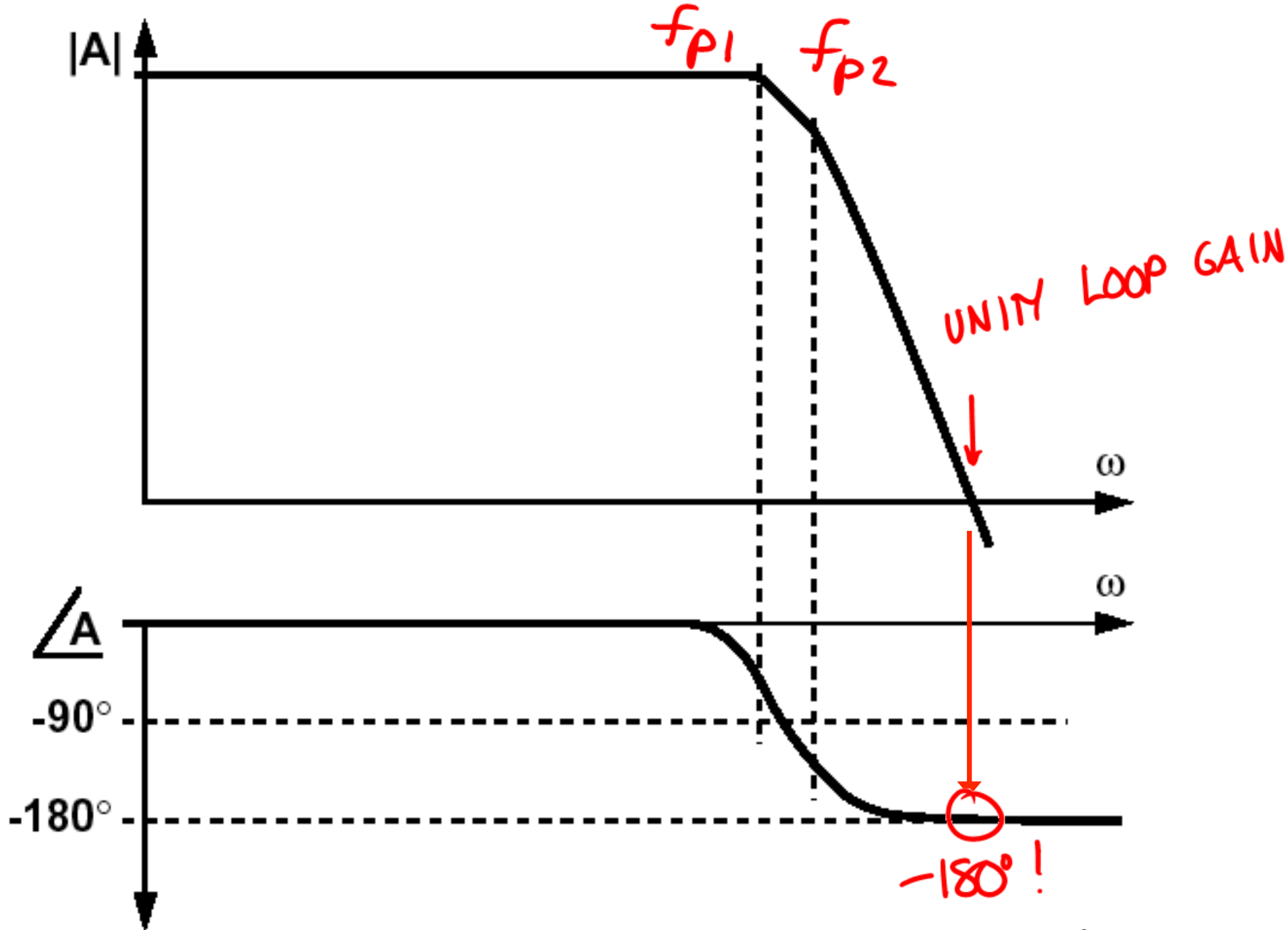
- **Easier to get from Bode plot**



Look at original Aβ for 2 stage op-amp

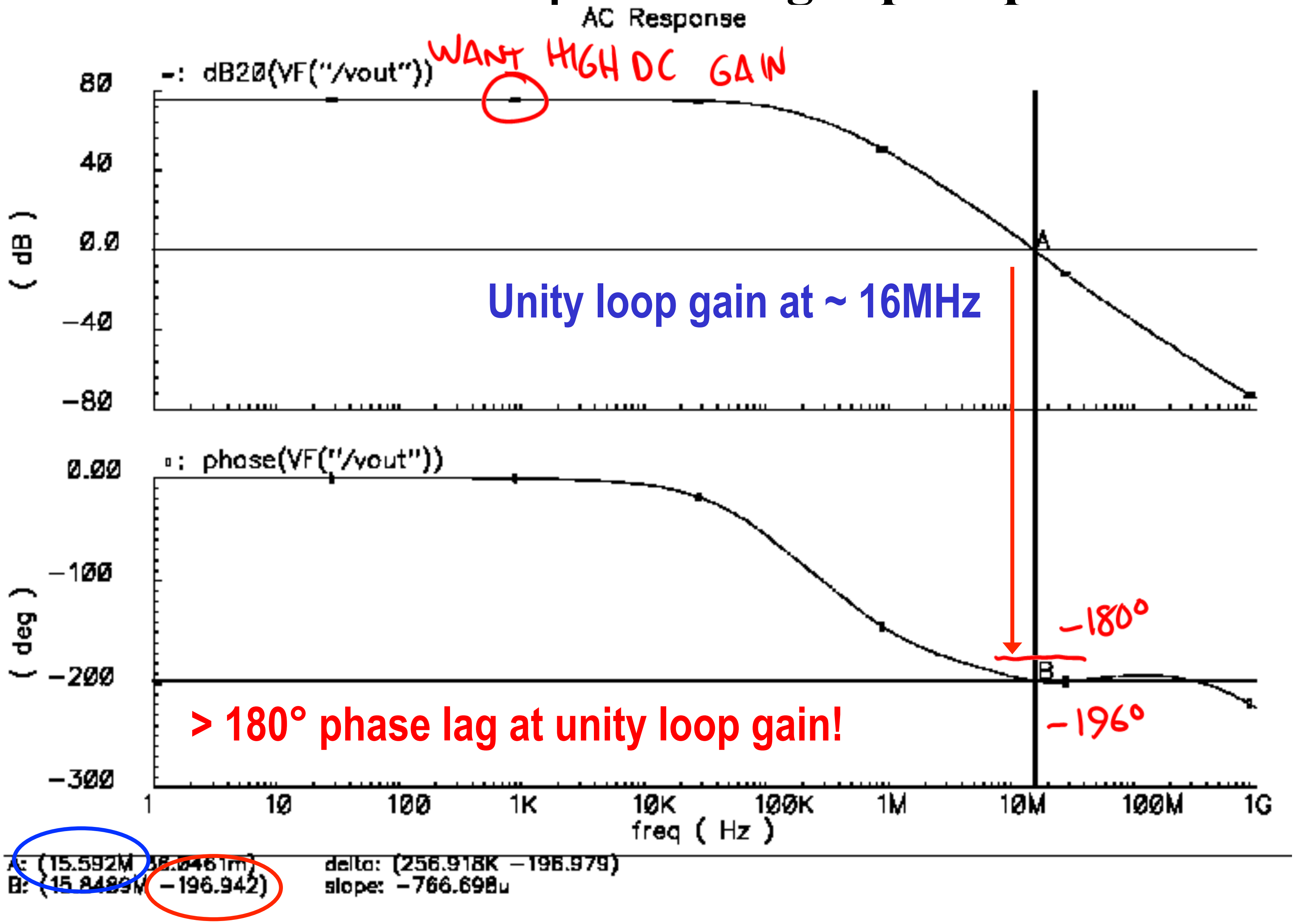
- **Find ω at which |Aβ| = 1; Check ∠Aβ -180° ?**

|AB| w B=1 FOLLOWER



Trouble!

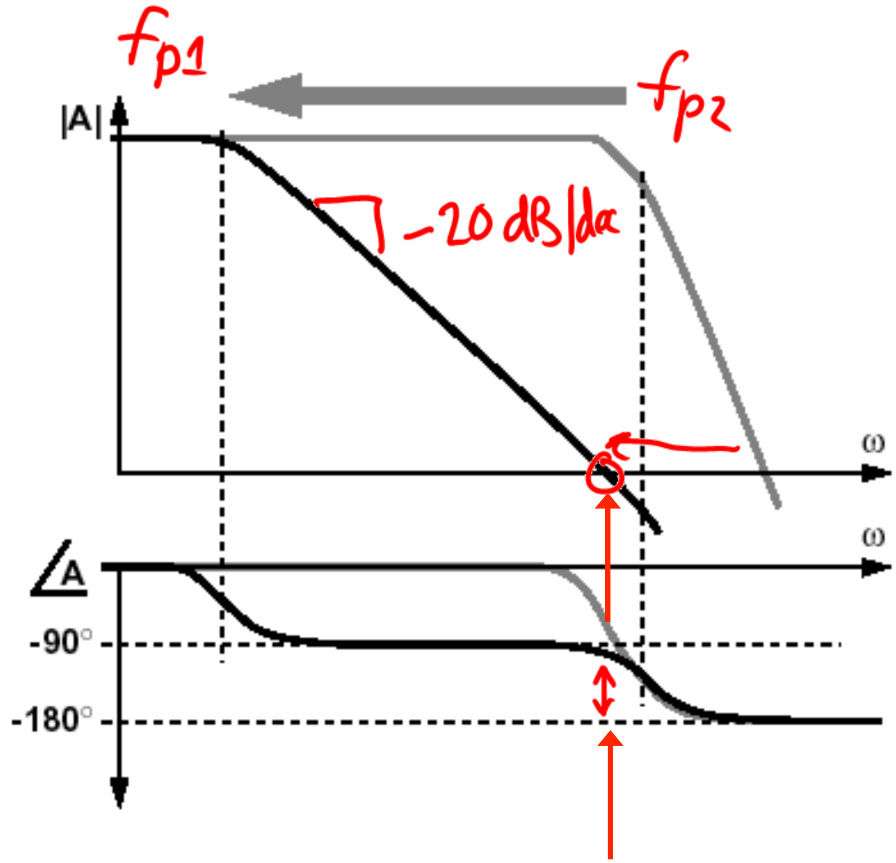
Simulation Aβ for 2 stage op-amp



- **Causes closed-loop instability**

Compensation: "Dominant Pole"

- **Move one pole to lower frequency**
- **How?**



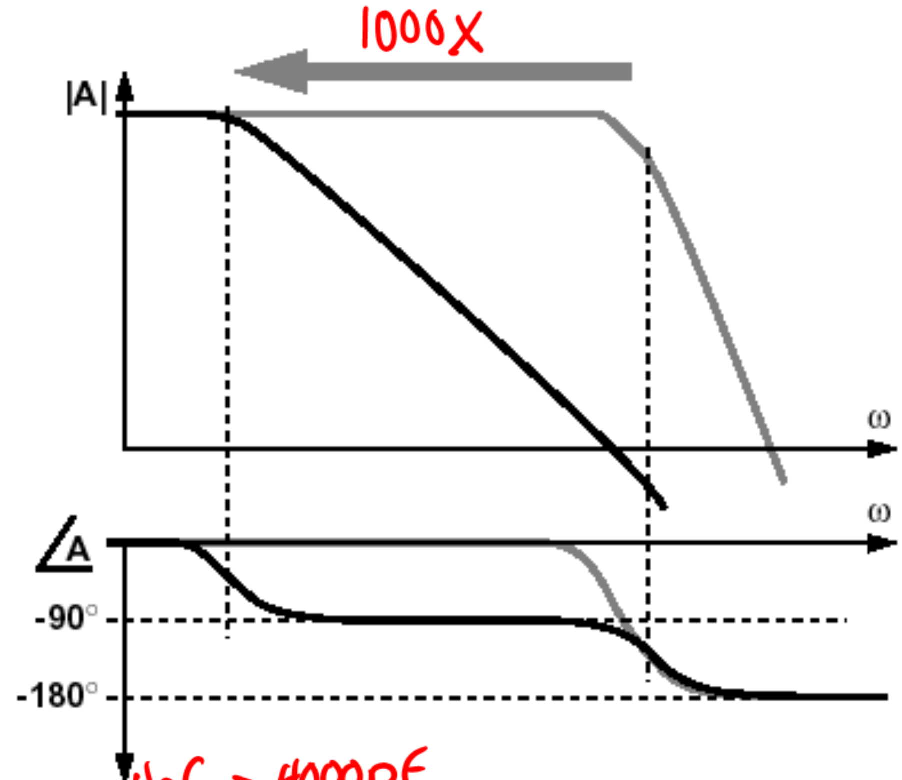
Move unity loop gain frequency f_T to lower value

So accumulated phase lag at f_T hasn't reached -180°

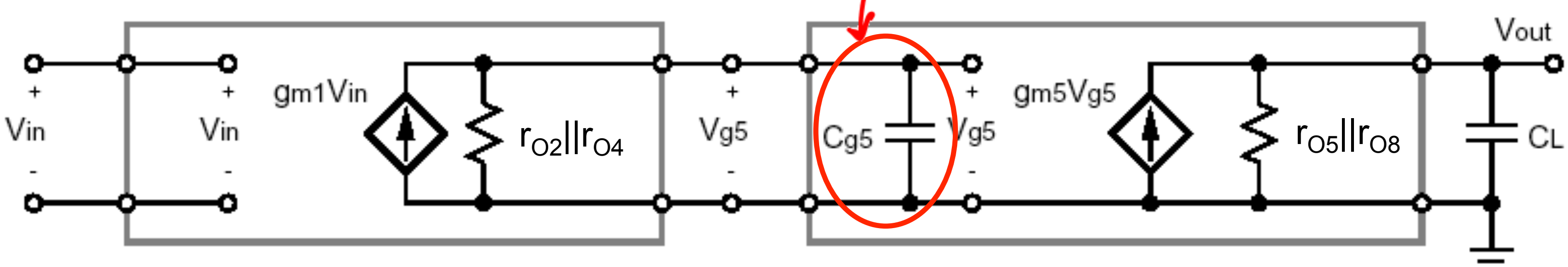
Compensation: "Dominant Pole"

- **Need to increase capacitance by ≈ 1000X:**

BAD! Die area cost

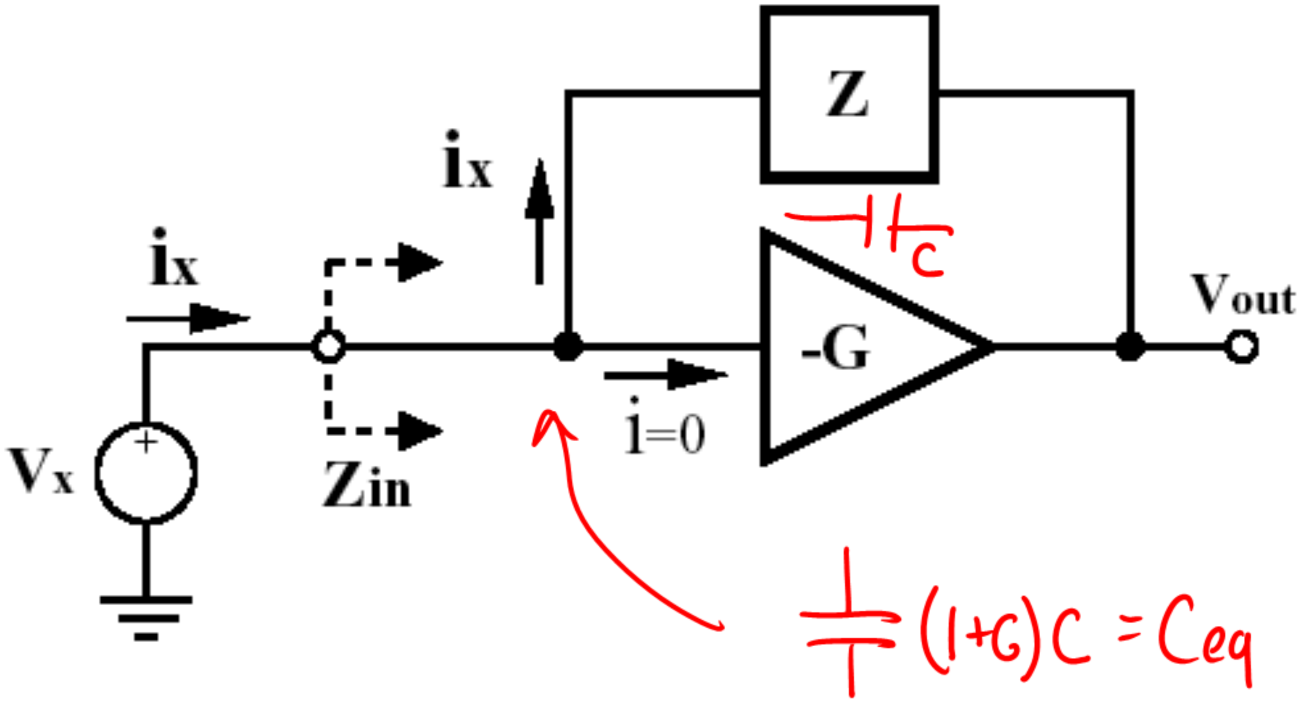


4pf → 4000pF



Miller Effect

- Impedance across inverting gain stage G
- Reduced by factor equal to (1+G)

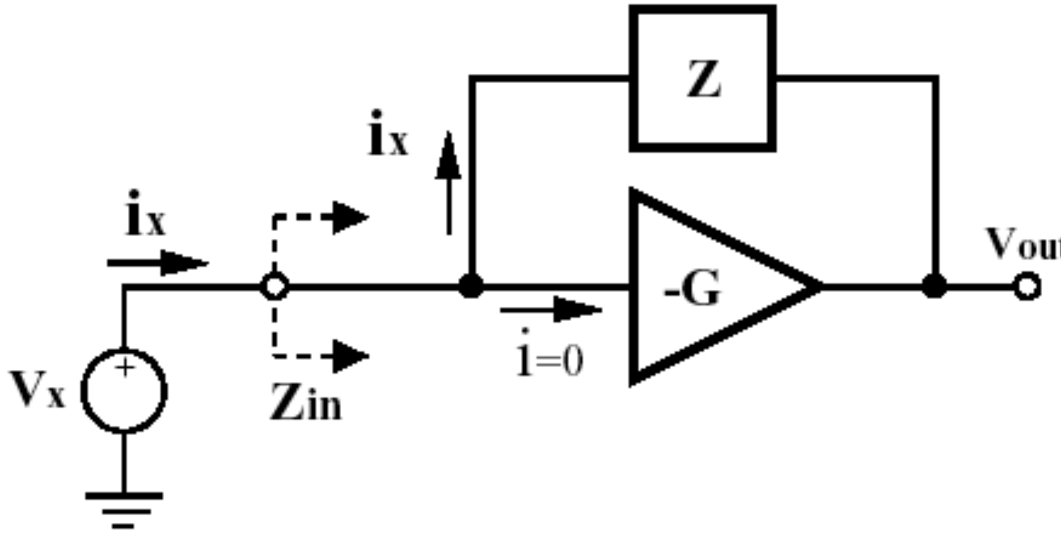


Math for Miller effect

$$i_x = \frac{v_x - (-Gv_x)}{Z}$$

$$i_x = \frac{v_x(1+G)}{Z}$$

$$\frac{v_x}{i_x} = Z_{in} = \frac{Z}{(1+G)}$$



- **Impedance across inverting gain stage G**
- **Reduced by factor equal to (1+G)**

Example: Impedance is capacitive

- **Capacitance multiplied by (1+G)**

$$Z_{in} = \frac{Z}{(1+G)}$$

$$Z = \frac{1}{sC} \quad \rightarrow \quad Z_{in} = \frac{1}{s\underbrace{(1+G)C}_{C_{eq}}}$$

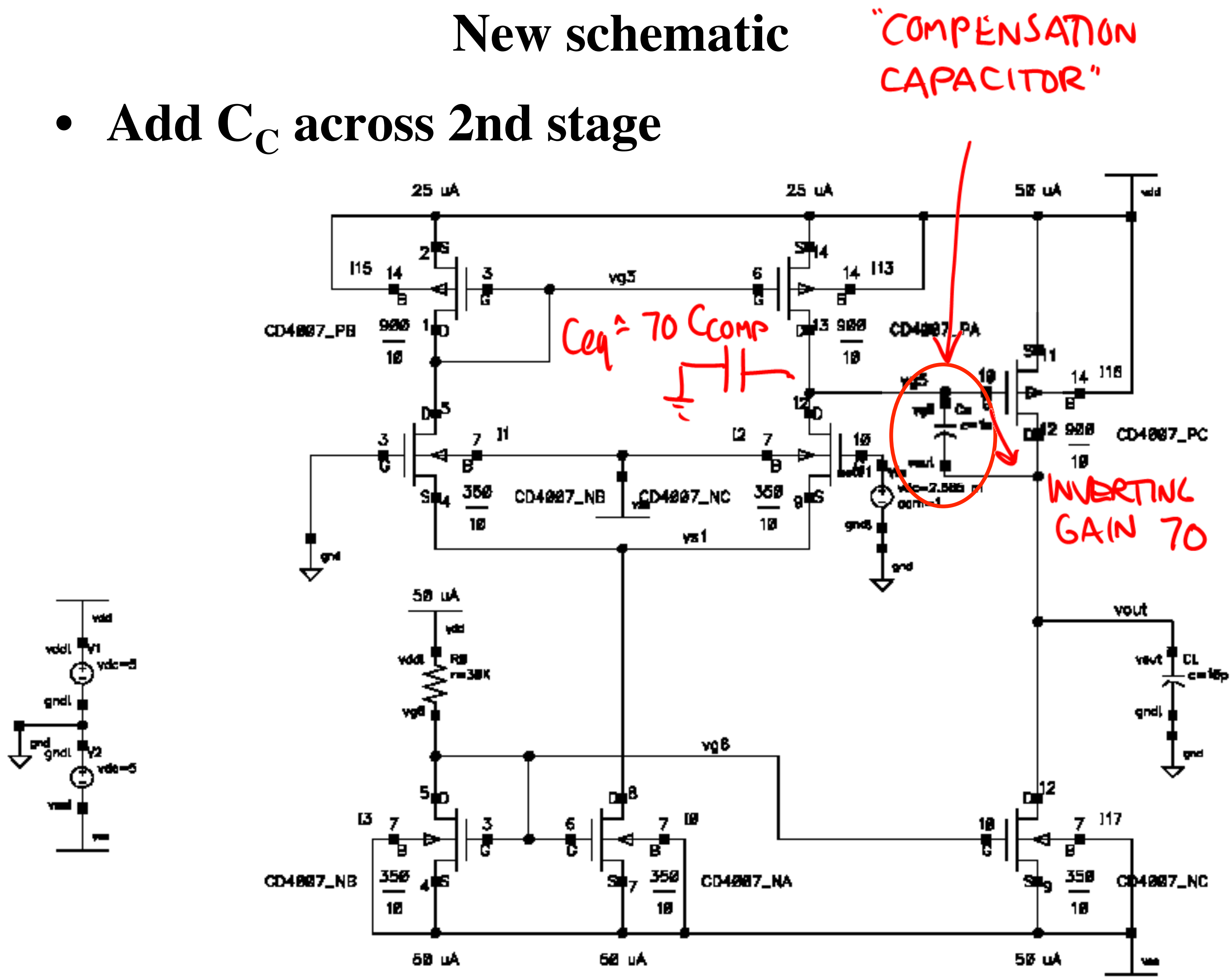
- **Equivalent capacitance higher by factor 1+G**
- **Problem for high bandwidth amplifiers**
- **Opportunity for compensation ...**

Miller Compensation

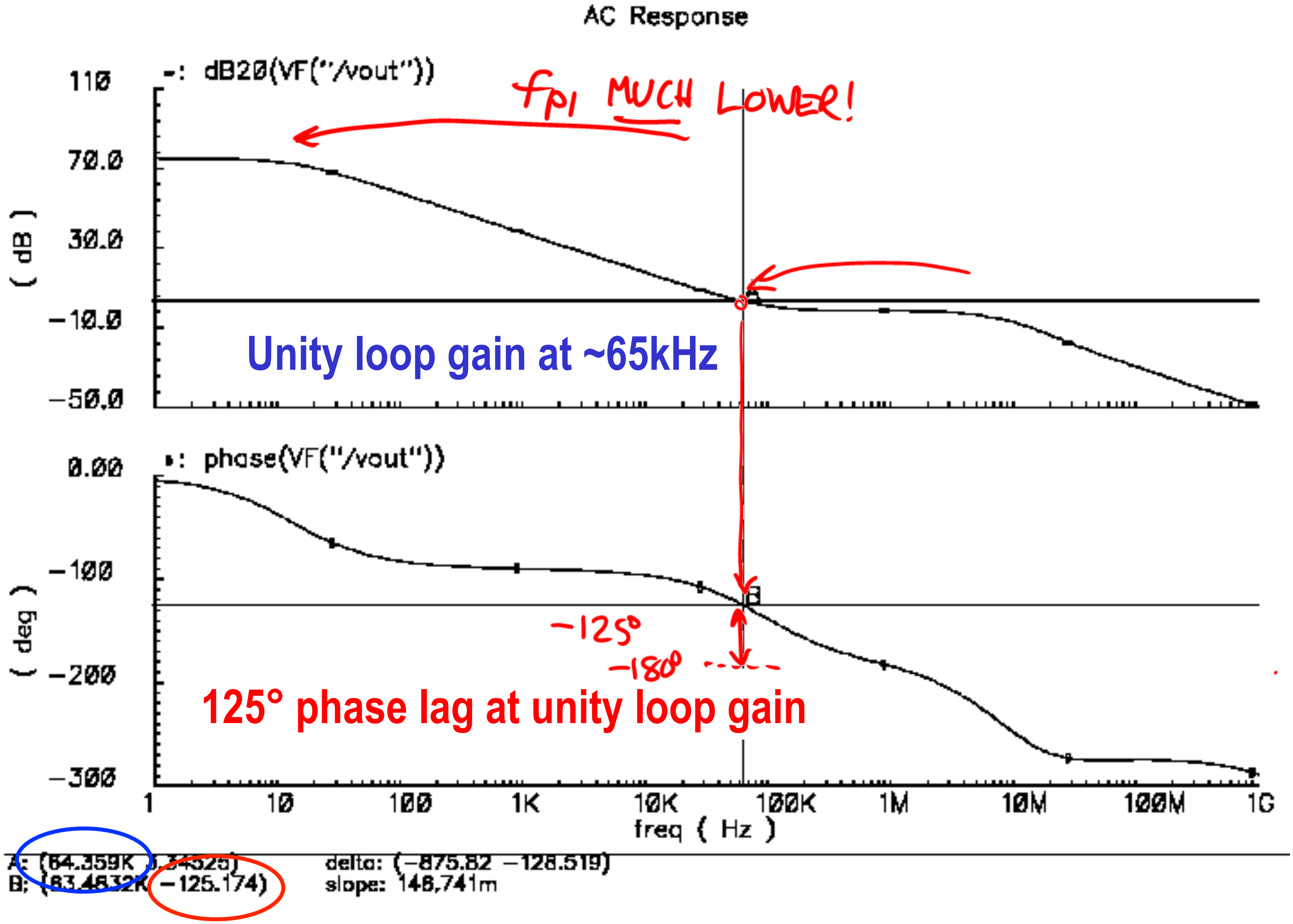
- Need effect of large capacitance
- Use Miller effect to multiply small on-chip capacitance to higher effective value
- Effect of large capacitance
  without die area cost of large capacitance

New schematic

- Add C_C across 2nd stage

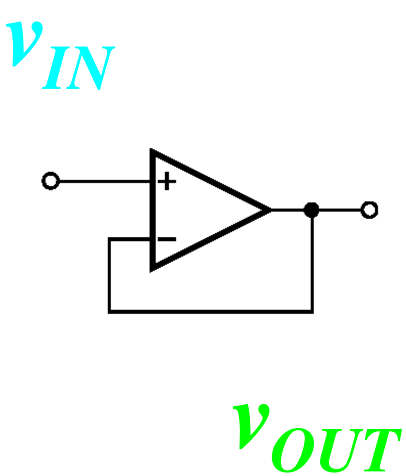


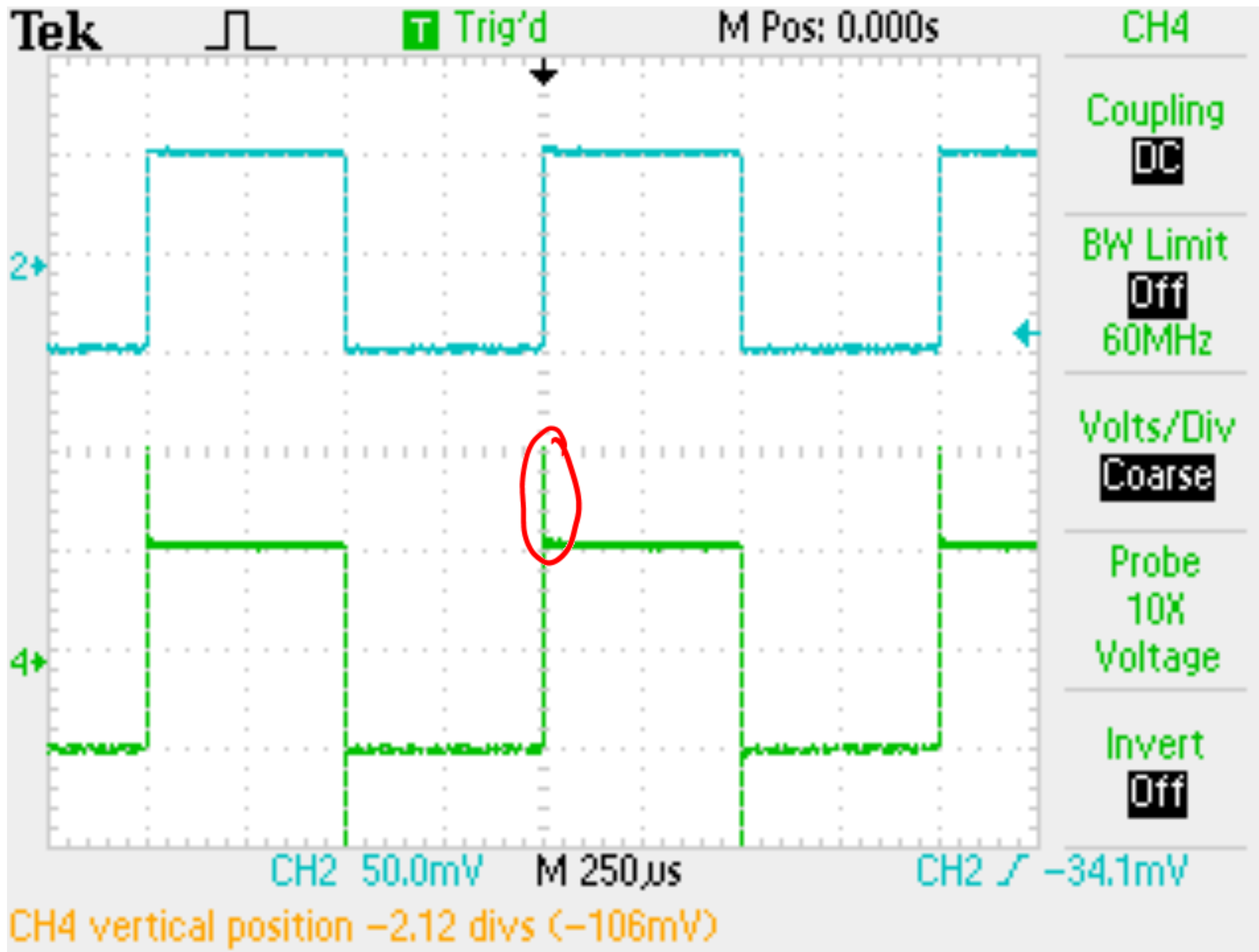
New loop gain transfer function



New step response

- **No oscillation!**

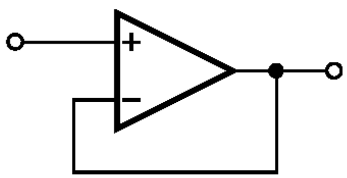




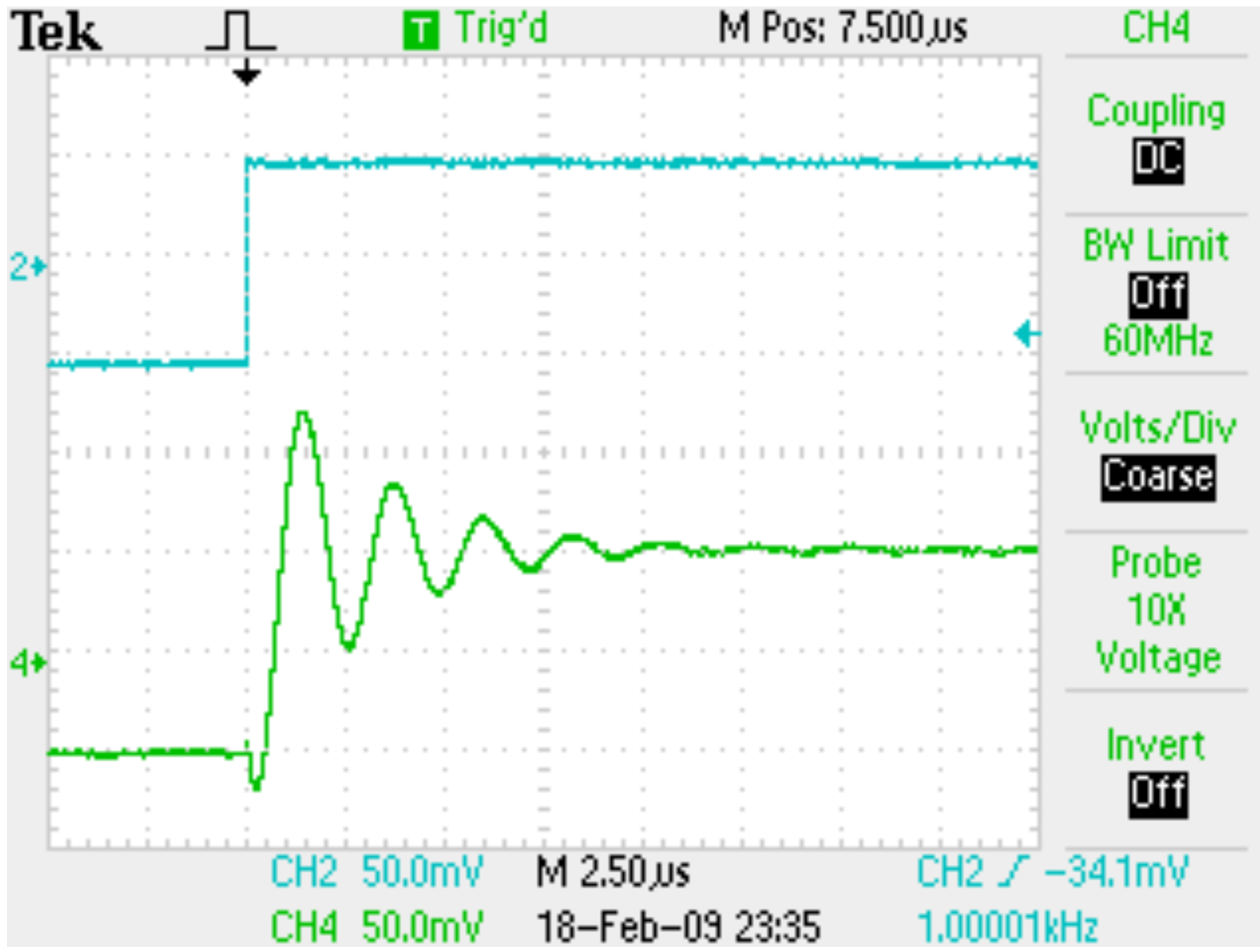
New step response with C_C

- **Zoom in on small-signal step response: Some overshoot and ringing**

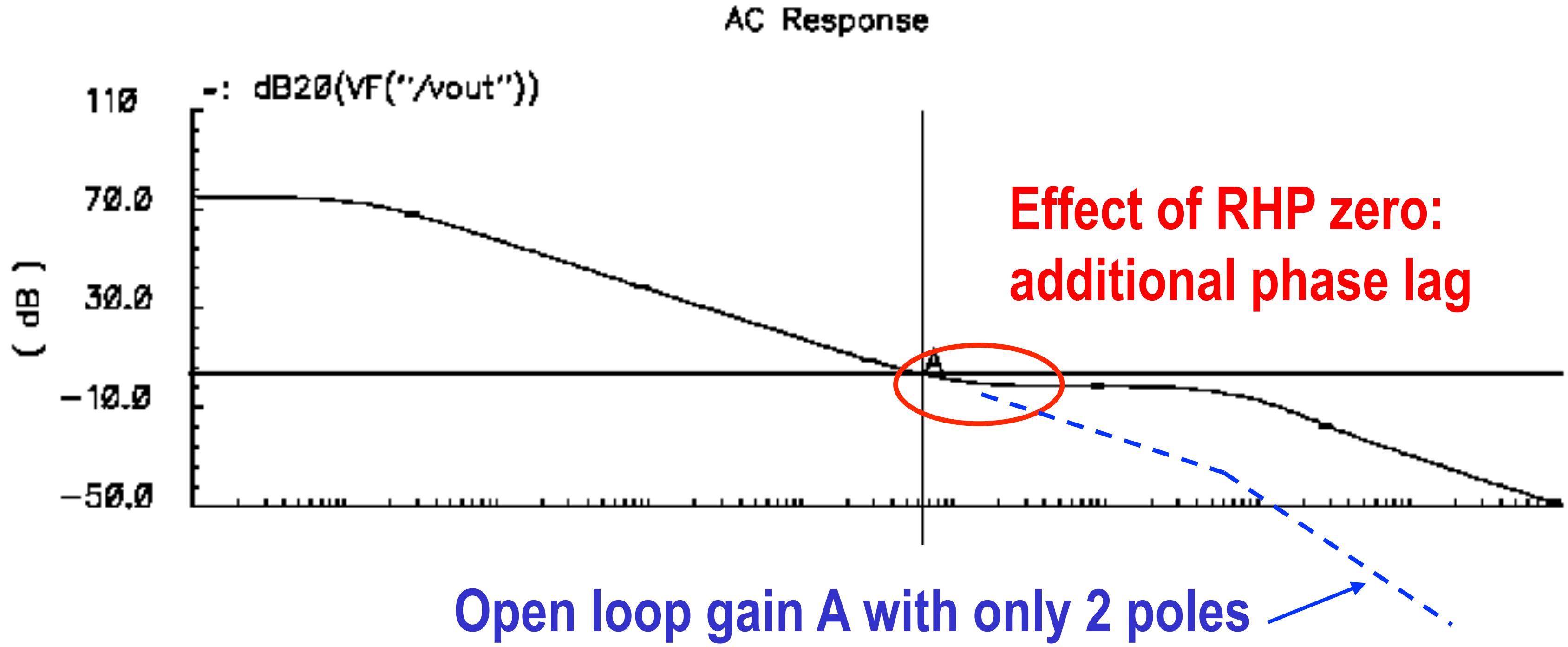
v_{IN}



v_{OUT}



Reason: RHP zero in complete transfer function



- **Complete transfer function looks like:**

$$A(j\omega) = \frac{A_0\left[1 - j\left(\omega/\omega_Z\right)\right]}{\left[1 + j\left(\omega/\omega_{p1}\right)\right]\left[1 + j\left(\omega/\omega_{p2}\right)\right]}$$

RHP ZERO NOT IN SIMPLE MODEL

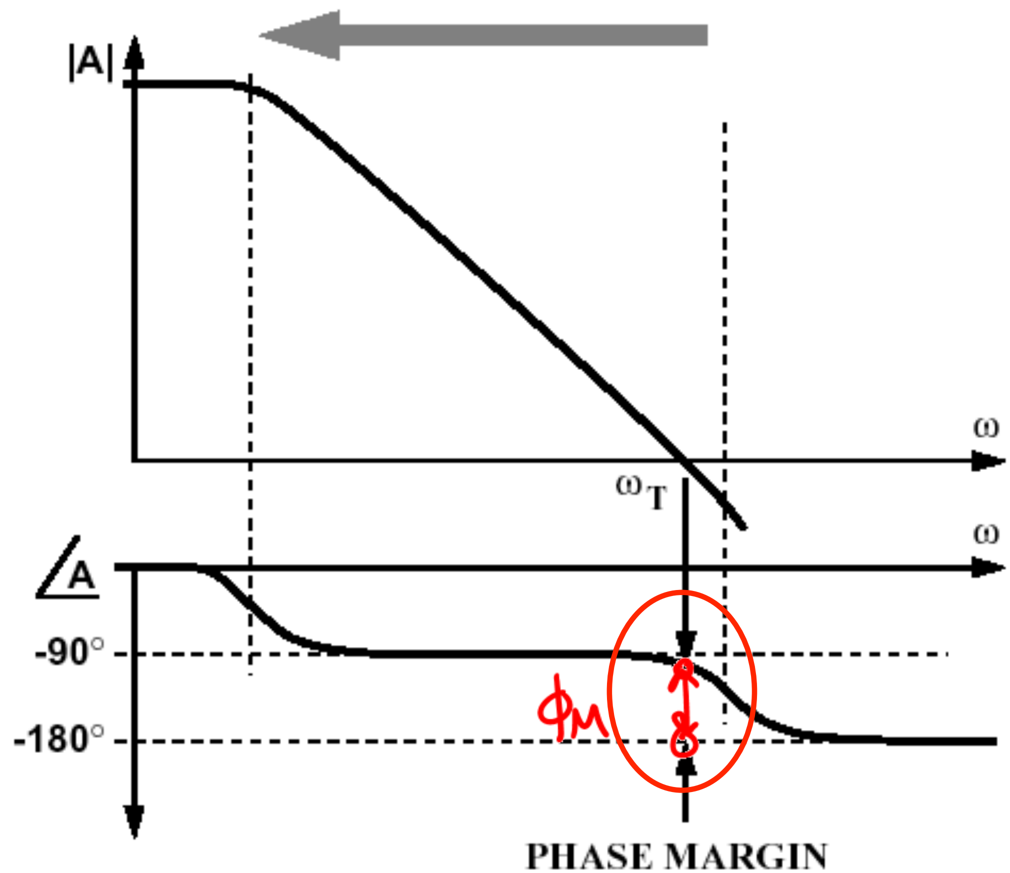
- **See Razavi 10.5, Johns & Martin 5.2**

"Phase margin"

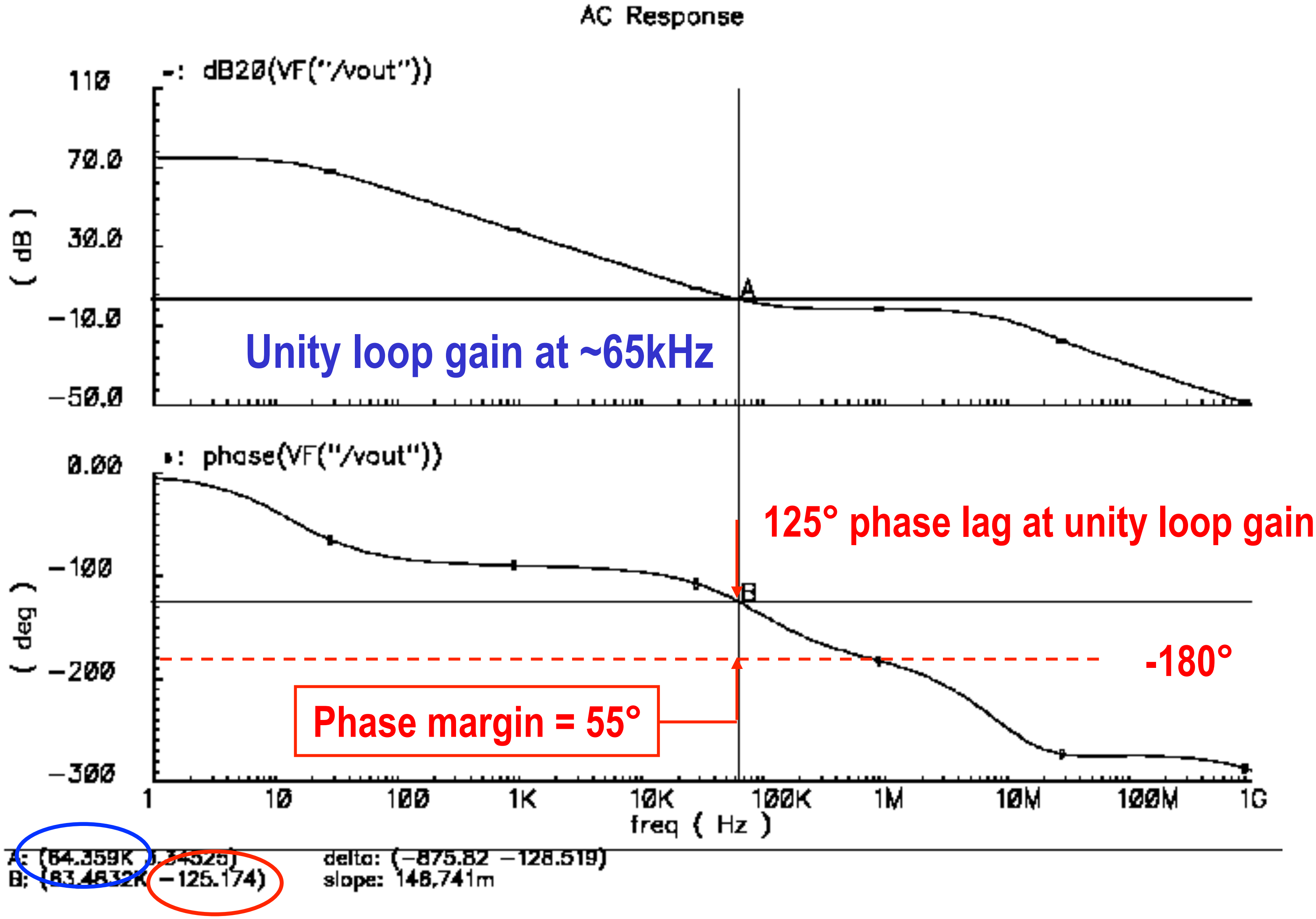
- **How stable is new transfer function?**
- **Phase margin =**

Phase lag at $|A\beta| = 1$ minus (-180°)

- **Usually want at least 60° for stable step response**

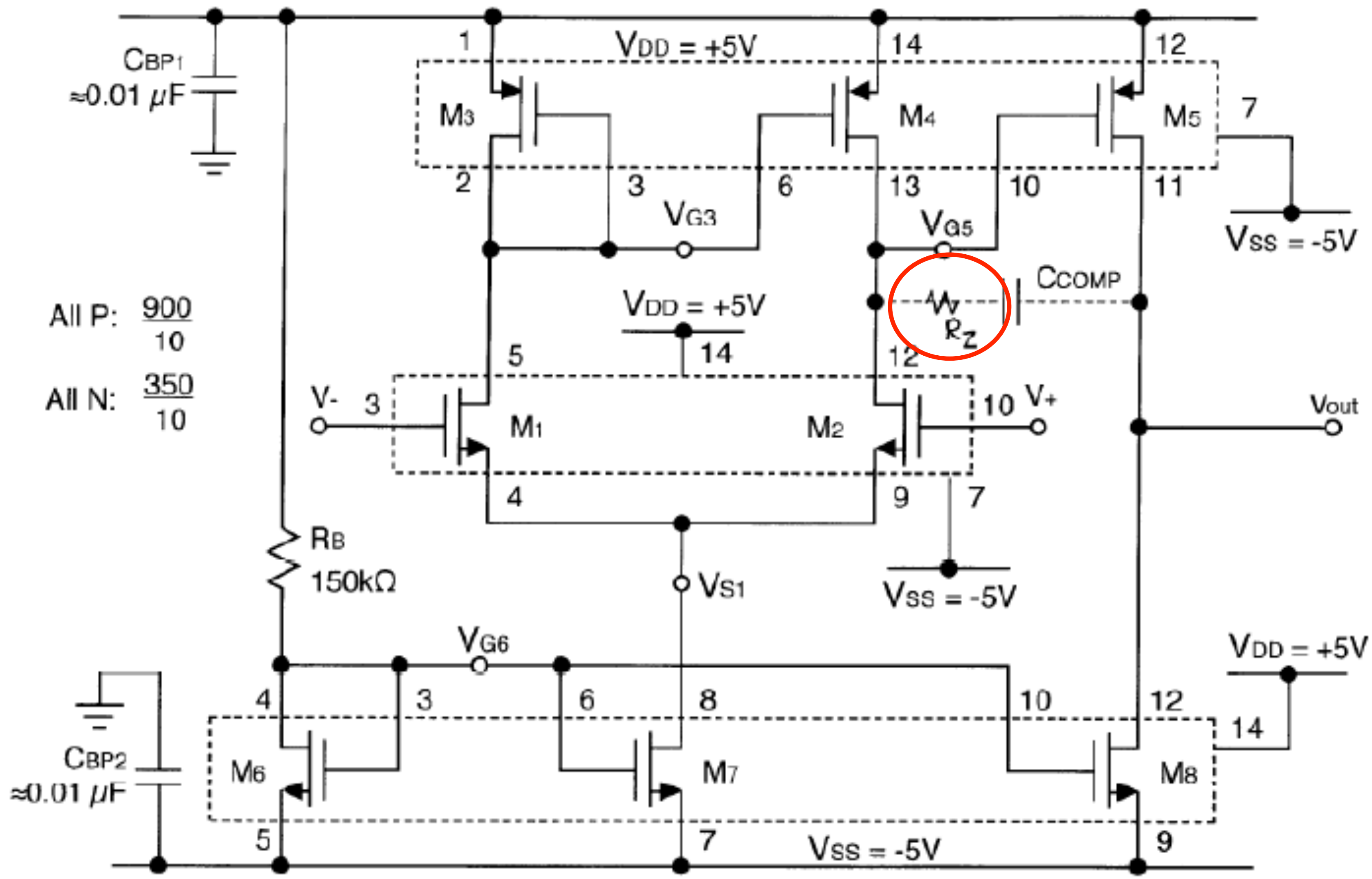


Phase margin of op-amp with C_C



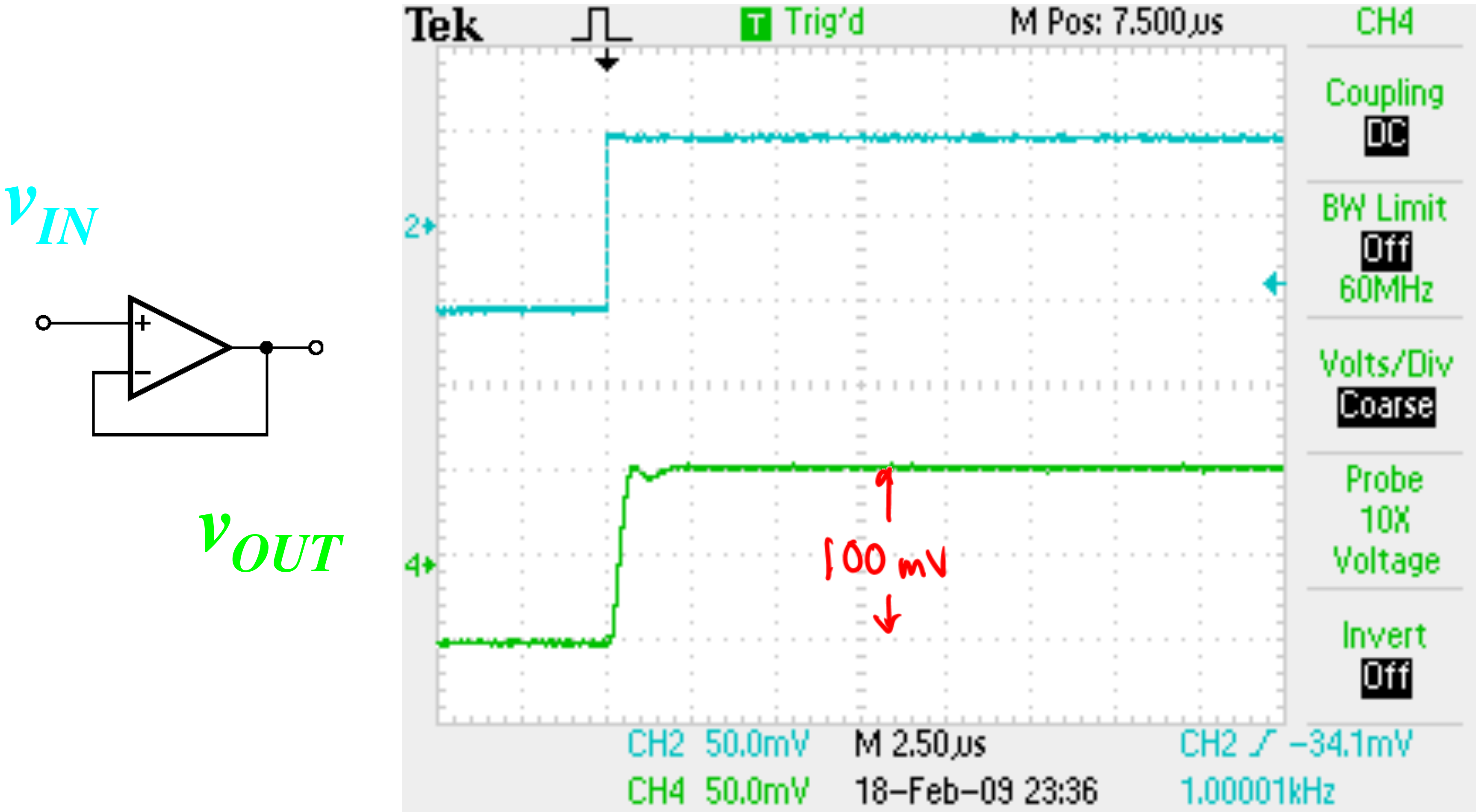
Solution to RHP zero problem

- **Add R_Z in series with C_C**
  Moves RHP zero to much higher frequency



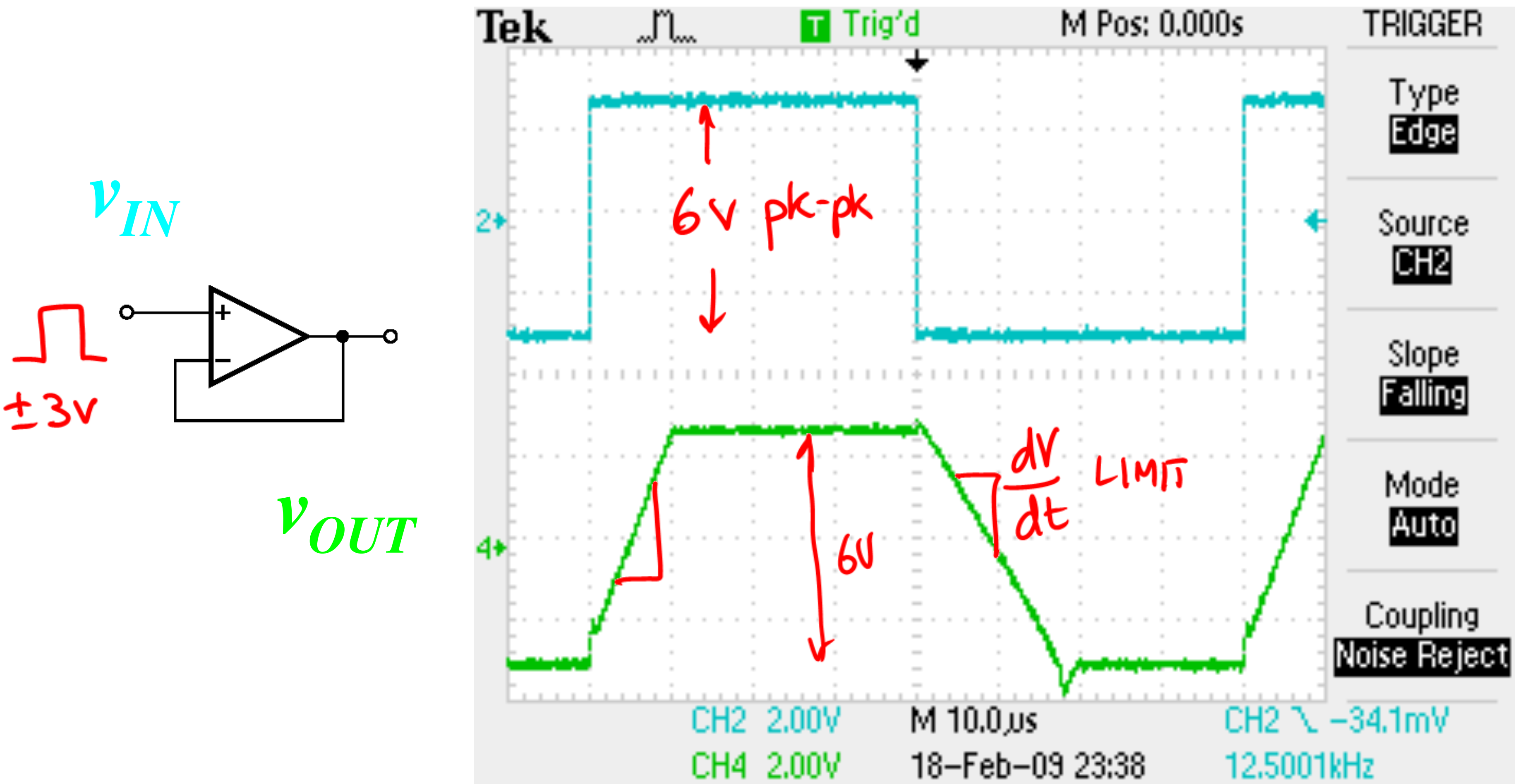
New step response with R_Z, C_C

- Zoom in on small-signal step response:
  No overshoot, ringing: phase margin improved



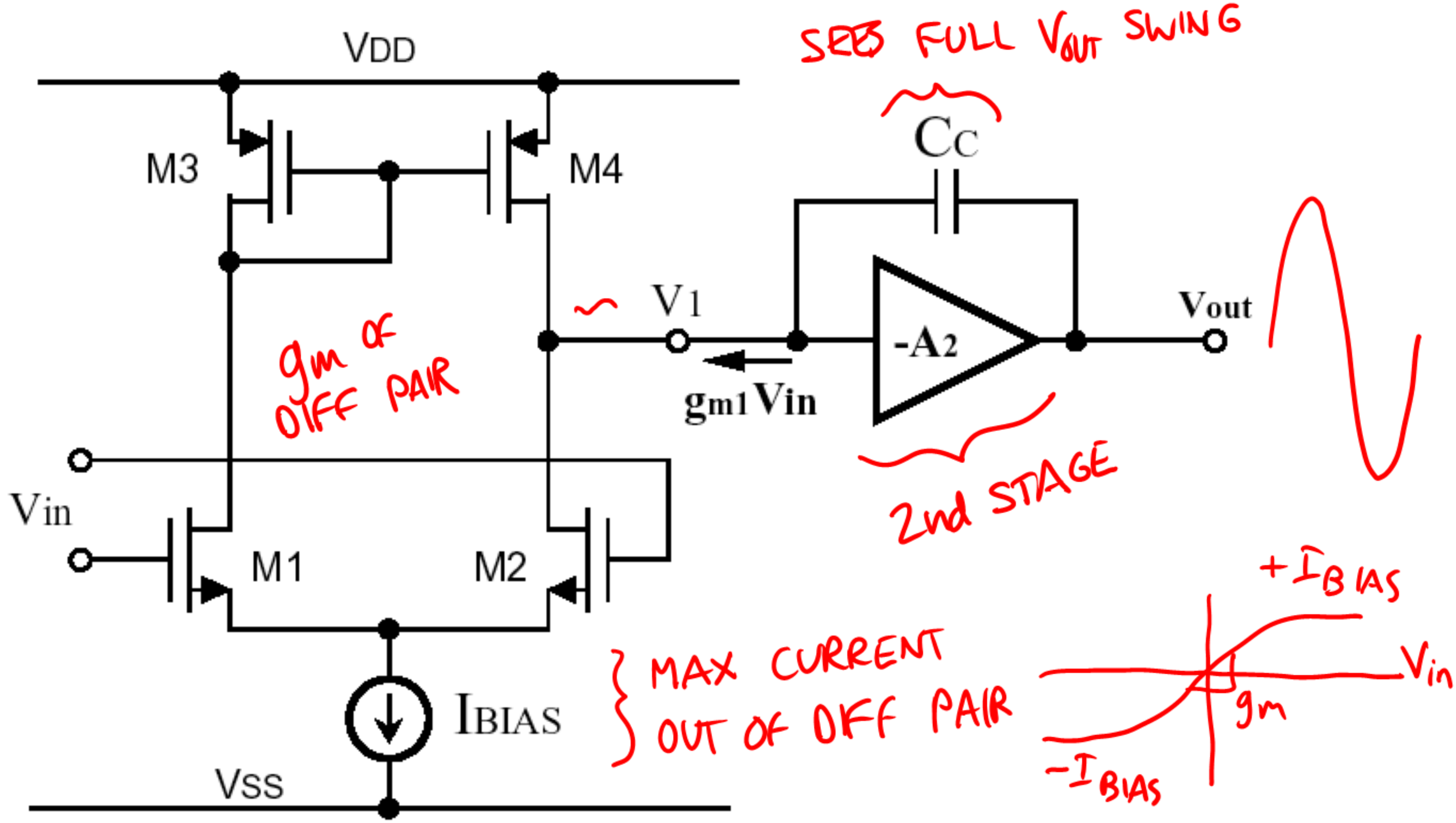
Large signal step response

- **Slew Rate Limiting!?!**



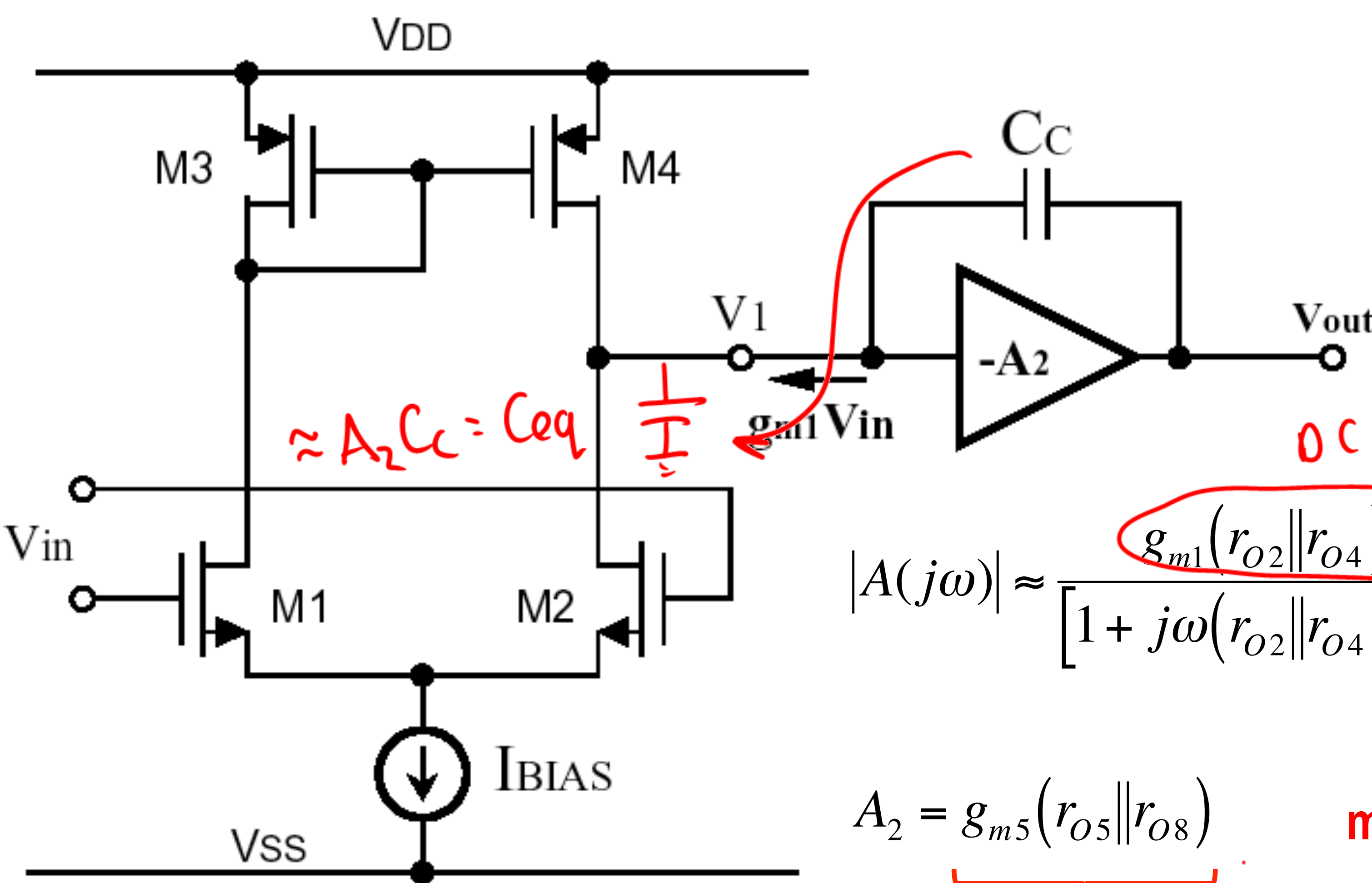
See Solomon op-amp paper for model; rising/falling asymmetry

Dominant pole op-amp model



- **Simpler model with dominant pole from C_C**

Approximate dominant pole transfer function



DC GAIN

$$|A(j\omega)| \approx \frac{g_{m1}(r_{O2}\|r_{O4})A_2}{\left[1 + j\omega(r_{O2}\|r_{O4})\underbrace{A_2C_C}_{\text{Miller multiplied } C_C}\right]}$$

$$A_2 = \underbrace{g_{m5}(r_{O5}\|r_{O8})}_{2^{nd}\text{ stage gain}}$$

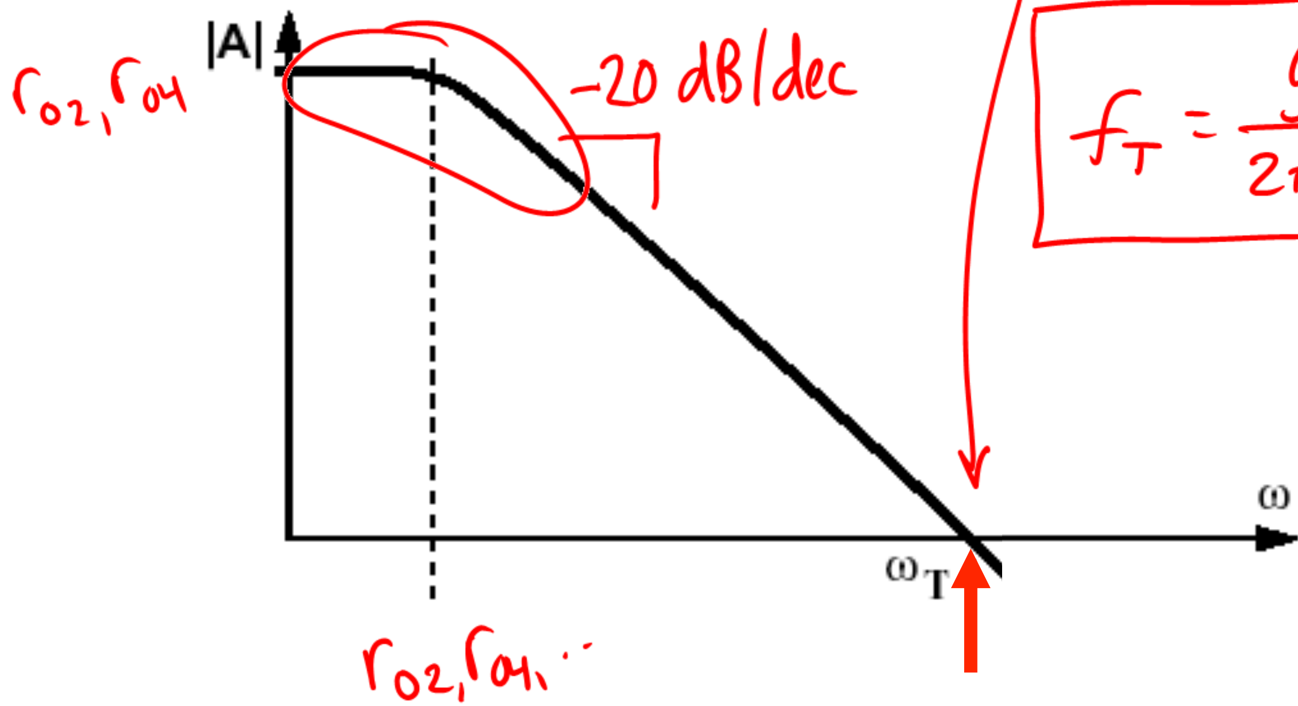
Unity gain frequency

- **Depends only on**
  - **Input stage transconductance g_{m1}**
  - **Compensation capacitor C_C**

$$|A(j\omega)| \approx \frac{g_{m1}(r_{O2}\|r_{O4})A_2}{[\omega(r_{O2}\|r_{O4})A_2C_C]}$$

$$|A(j\omega)| = 1 \quad at \quad \omega_T$$

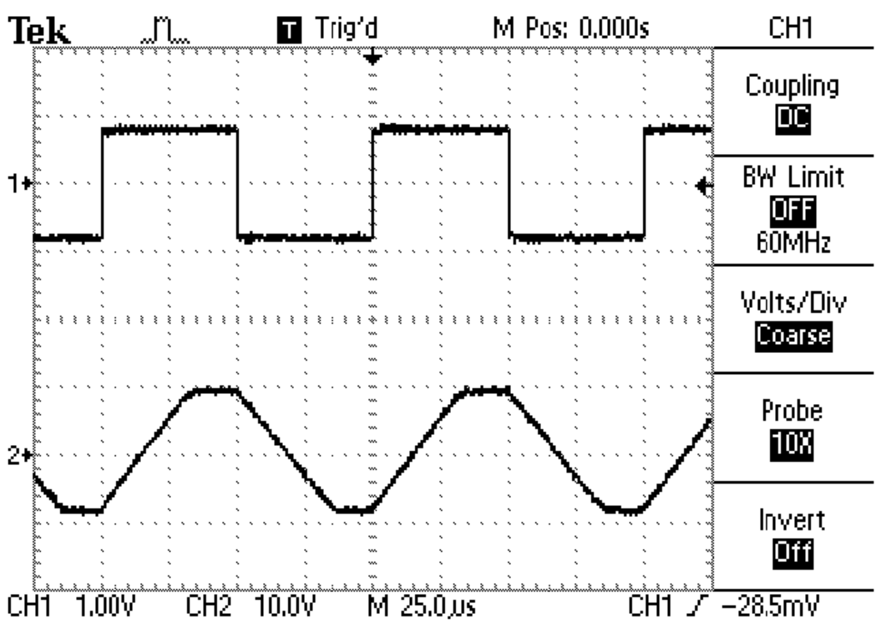
$$\omega_T \approx \frac{g_{m1}}{C_C}$$

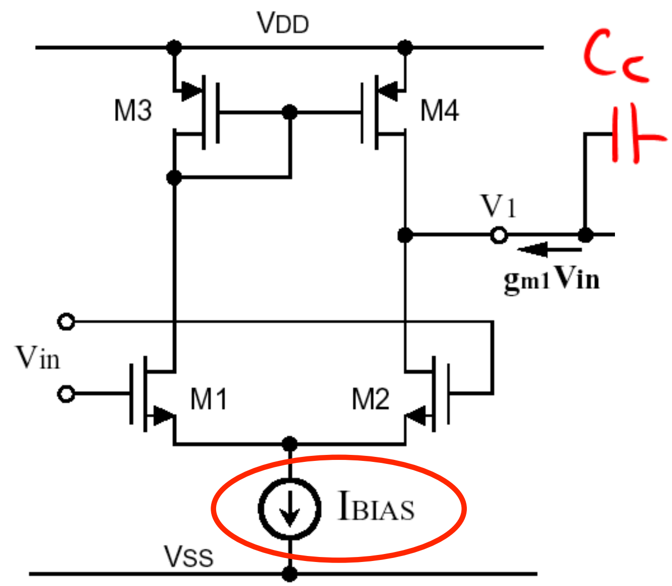


$$\boxed{f_T = \frac{g_{m1}}{2\pi C_C}}$$

Slew rate

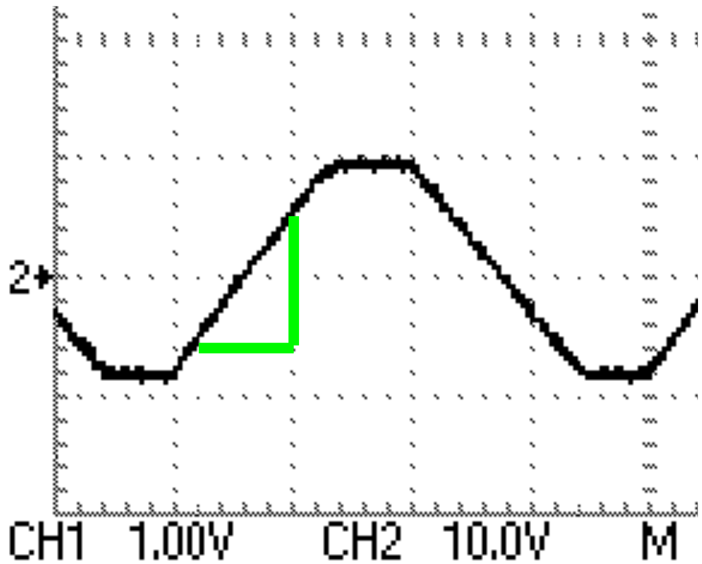
- I= C dV/dt
- Only limited current I_{BIAS} available to charge, discharge C_C

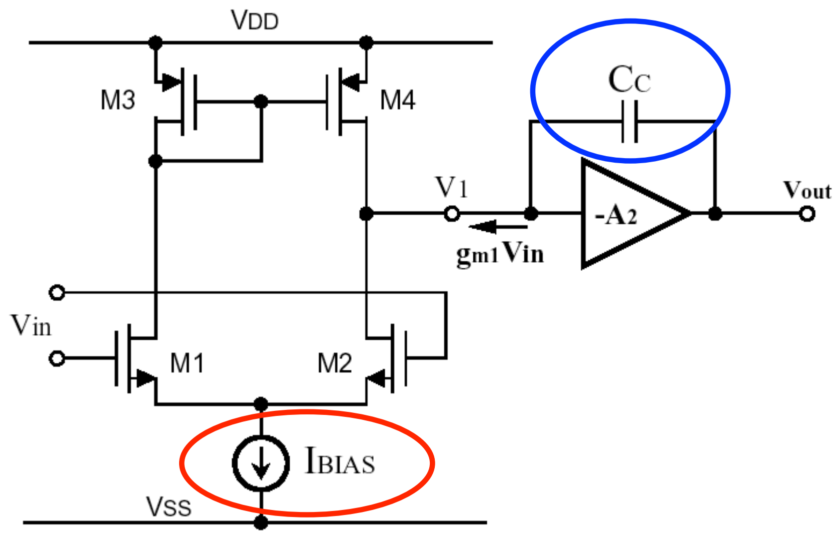


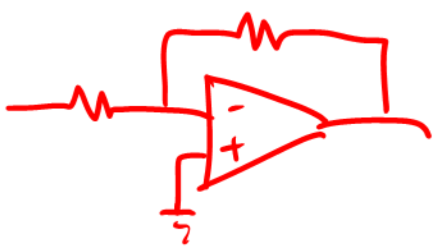


Slew rate

- I= C dV/dt $\Rightarrow$ $$\frac{dV}{dt} = \frac{I_{BIAS}}{C_C}$$







Summary Op-amp:

- Stability
- Compensation
- Miller effect
- Phase Margin
- Unity gain frequency
- Slew Rate Limiting